MATHEMATICS

Grade 7

Student Textbook

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Table of Contents

UNIT 1 RATIONAL NUMBERS

Unit Outcomes:

After Completing this unit, you should be able to:

- define and represent rational numbers as fractons.
- show the relationship among $\mathbb{W}$, $\mathbb{Z}$ and $\mathbb{Q}$.
- order rational numbers.
- Perform operation with rational numbers.

Introduction

In the previous grades you had already learnt about fractions and decimals. These numbers together with integers form a bigger set of numbers known as the set of rational numbers. In this unit you will learn about rational unmbes and their basic properties. You will also learn how to perform the four fundamental operations on rational numbers.

1.1. The Concept of Rational Numbers

Group work 1.1

Discuss with your friends/ partners.

The Venn diagram below shows the relationships between the sets of Natural numbers, whole numbers and Integers.

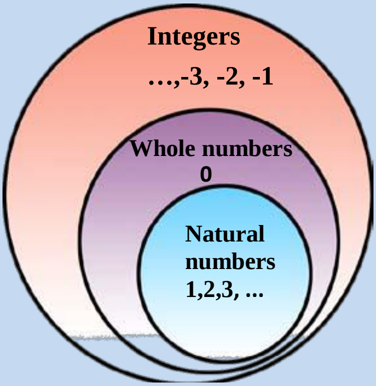


Figure 1.1 Relationship among ℕ, 𝕎 and ℤ

1. Define the set of:
   a. Natural numbrs
   b. Whole numbers
   c. Integers
2. Name six natural numbers.
3. Name six whole numbers.
4. Name six positive integers.
5. Name six negative integers.
6. To which set(s) of numbers does each of the following numbers belong?

   a. 0 b. 25 c. -102 d. $\frac{30}{2}$
7. Put < or > in stead of the box between each pairs of numbers given below to make it true.

   a. -76 $\square$ 600 b. -1200 $\square$ -800 c. 0 $\square$ -10,000

1.1.1. Revision on Integers

In grades 5 and 6 mathematics lessons you have already learnt several facts about the sets of natural numbers (ℕ), whole numbers, (𝕎) and integers (ℤ). In this subsection you will revise some important facts about the set of integers.

Activity 1.1

Discuss with your friends/ partners

1. For each of the following statements write "true" if the statement is correct or "false" other wise.
   (Hint: ∪ = union and ∩= intersection).
   a. The set {0, 1, 2, 3, …} describes the set of natural numbers.
   b. The set {…, -2, -1, 0, 1, 2, …} describes the set of integers.
   c. ℕ ∪ 𝕎 = {0, 1, 2, 3, …}.
   d. ℕ ∩ ℤ = {1, 2, 3, 4, …}.
   e. 126 is a natural number.

2. a. Is every natural number a whole number? If it is so, can you say $\mathbb{N} \subseteq \mathbb{W}$?

b. Is every natural number an integer? If so, can you say $\mathbb{N} \subseteq \mathbb{Z}$?

c. Is every whole number an integer? If so, can you say $\mathbb{W} \subseteq \mathbb{Z}$?

Note: The set of numbers consisting of whole numbers and negative numbes is called the set of integers. The set of integers is dentoed Z = {..., -3, -2, -1, 0, 1, 2, 3 ,...}.

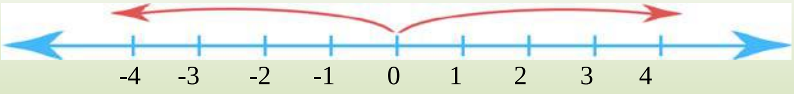


Figure 1.2 Number line

Figure 1.3 Anders Celsius

- Anders Celsius the Swedish astronomer who lived between 1701 and 1744 A.D. He devised a way of measuring temperature which was adjusted and improved after his death.

Directed numbers are used in telling the temperature in degree Celsicus' (^{o}C). Thus if the temperature is 20 degree Celsius above zero, you can read as **positive** twenty degree Celsius (+20 ^{o}C) and the temperature is (-20) degree Celsius below zero you can read also **negative twenty** degree celsius (-20 ^{o}C).

Example 1: Give the directed number describing each of the following temperatures.

a. Seventy five above zero.

b. Forty below zero.

c. Twenty five below zero.

d. Twenty one above zero.

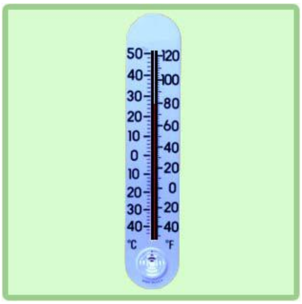


Figure 1.4 Thermometer

Solution

a. Positive seventy five (+75).
b. Negative forty (-40).
c. Negative twenty five (-25).
d. Positive twenty one (+21).

From Grad 5 and 6 Mathematics lesson recall that:

- ✓ The set of natural numbers, denoted by $\mathbb{N}$ is described by $\mathbb{N}= \{1, 2, 3, \ldots\}$.
- ✓ The set of whole numbers, denoted by $\mathbb{W}$ is described by $\mathbb{W} = \{0,1,2,3,\ldots\}$.
- ✓ The set of integers, denoted by $\mathbb{Z}$ is described by $\mathbb{Z} = \{\ldots,-2, -1, 0, 1, 2, \ldots\}$.
- ✓ The sum of two natural numbers is always a natural number.
- ✓ The product of two natural numbers is always a natural number.
- ✓ The difference and quotient of two natural numbers are not always natural numbers.
- ✓ The sum of two whole numbers is always a whole numbers.
- ✓ The product of two whole numbers is always a whole number.
- ✓ The difference and quotient of two whole numbers are not always whole number.
- ✓ The sum of two integers is always an integer.
- ✓ The product of two integers is always an integer.
- ✓ The difference of two integers is always an integer.
- ✓ The qoutient of two integers is not always an integer.

1.1.2. Revision of Fractions

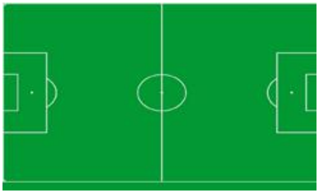
From grade 5 and 6 mathematics lessons, you have learnet about definition of fractions, operations on fractions and types of fractions. Recall the following:

Note: Fractions are numbers of the form $\frac{a}{b}$ where $\frac{a}{b} = a \div b$ when a and b are whole numbers and b is not equal to zero ($b \neq 0$).

In the fraction $\frac{a}{b}$,the numerator is 'a' and the denominator is 'b' .

Example 2: Examples of fractions in Figure 1.5 below.

This foot ball pitch has two halves

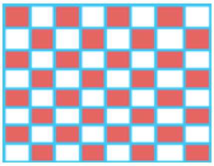


One part is one half or $\frac{1}{2}$ of the pitch

This DVD has eight equal Sectors

The shaded part is $\frac{7}{8}$ of the DVD

This chess board has 64 equal small squares



One part is one sixty – fourth or $\frac{1}{64}$ of the chess board

Figure 1.5 Examples of fractions

Example 3: Let two fivths or $\frac{2}{5}$ of the parking spaces be occupied, then find the numerator and denominator.

Figure 1.6 Cars

Solution:

$\frac{2}{5}$

The top number is called the **numerator.**

The bottom number is called the **denominator.**

Note: Based on the numerator and denominator, you can classify a given fraction into two types. These are:

i. Proper fraction
ii. Improper fraction

✓ If the numerator of a fraction is less than its denominator, then the fraction is a proper fraction. That is the fraction $\frac{a}{b}$ is called proper fraction, if $a < b$.

- ✓ If the numerator of a fraction is greater than or equal to its denominator, then the fraction is an improper fraction. That is the fraction $\frac{a}{b}$ is called improper fraction, if $a \geq b$.
- ✓ If an improper fraction is expressed as a whole number and proper fraction, then it is called Mixed number.

Activity 1.2

Discuss with your teacher orally

1. Name the numerator and denominator of each fraction.

   a. $\frac{5}{6}$ b. $\frac{12}{10}$ c. $3\frac{2}{5}$ d. $\frac{a}{b}$ where $b \neq 0$
2. Give examples of your own for proper fractions, improper fractions and mixed numbers.
3. Change these improper fractions to mixed numbers.

   a. $\frac{5}{2}$ b. $\frac{39}{4}$ c. $\frac{26}{9}$ d. $\frac{17}{10}$
4. Change these mixed numbers to improper fractions.

   a. $3\frac{1}{4}$ b. $4\frac{2}{5}$ c. $3\frac{7}{10}$ d. $1\frac{9}{100}$

1.1.3. Revision on Equivalent Fractions

Activity 1.3

Discuss with your teacher orally

1. Copy and complete each set of equivalent fractions.

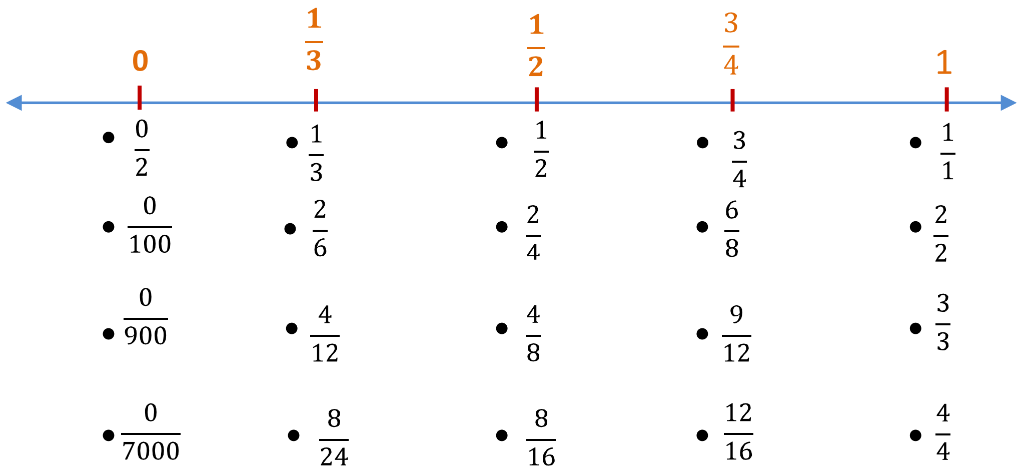
   a. $\frac{3}{4} = \frac{\quad}{8} = \frac{\quad}{12} = \frac{\quad}{16} = \frac{\quad}{20} = \frac{\quad}{24}$.

   b. $\frac{2}{7} = \frac{\quad}{14} = \frac{\quad}{21} = \frac{\quad}{28} = \frac{\quad}{35} = \frac{\quad}{42}$.

   c. $\frac{4}{5} = \frac{\quad}{10} = \frac{\quad}{15} = \frac{\quad}{20} = \frac{\quad}{25} = \frac{\quad}{30}$.
2. Consider the given fractions $\frac{6}{8}, \frac{12}{16}, \frac{24}{32}$ and $\frac{48}{64}$, are they equivalent fractions?

An interesting property of rational numbers is that infinitely many different fractions may be used to represent the same rational numbers. Figure 1.7 below

shows that a point on the number line can be represented by infinitely many different fractions. For example $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}, \frac{8}{16}$ all represent the same point $\frac{1}{2}$.



Finguer 1.7 Number line

Note: From the above Figure 1.7 we get:

i. $\frac{0}{2} = \frac{0}{100} = \frac{0}{900} = \frac{0}{7000} = 0$, **That is** $\frac{0}{2}$, $\frac{0}{100}$, $\frac{0}{900}$ **and** $\frac{0}{7000}$ **are equivalent fractions.**

ii. $\frac{1}{3} = \frac{2}{6} = \frac{4}{12} = \frac{8}{24} = \frac{1}{3}$ **that is** $\frac{1}{3}, \frac{2}{6}, \frac{4}{12}$ **and** $\frac{8}{24}$ **are equivalent fractions.**

iii. $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \frac{1}{2}$ **that is** $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}$ **and** $\frac{8}{16}$ **are equivalent fractions.**

iv. $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{3}{4}$ **that is** $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}$ **and** $\frac{12}{16}$ **are equivalent fractions.**

v. $\frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = 1$ **that is** $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}$ **and** $\frac{4}{4}$ **are equivalent fractions.**

Therefore, using the above discussion, we define equivalent fractions as follows:

Definition 1.1. Fractions that represent the same point on the number line are called Equivalent Fractions.

Example 4: $\frac{1}{3}, \frac{3}{9}, \frac{9}{27}, \frac{27}{81}$ and $\frac{81}{243}$ are equivalent fraction. You may observe that:

$$\frac{3}{9}=\frac{3\times1}{3\times3}, \frac{9}{27}=\frac{9\times1}{9\times3}, \frac{27}{81}=\frac{27\times1}{27\times3} \text{ and } \frac{81}{243}=\frac{81\times1}{81\times3}.$$

Further more the above example 4 can be generalized by the fundamental properties of fraction as follows:

Fundamental properties of fraction:

For any fraction $\frac{a}{b}$ if m is any number other than zero, $\frac{a}{b} = \frac{a}{b} \times \frac{m}{m}$. Therefore, $\frac{a}{b}$ and $\frac{a}{b} \times \frac{m}{m}$ are equivalent fractions.

Note: Two fractions $\frac{a}{b}$ and $\frac{c}{d}$, b, d $\neq 0$ are equivalent if and only if a×d = b×c.

Equivalenetly $\frac{a}{b} = \frac{c}{d}$ if and only if a×d = b×c.

Look at the following example very carefully.

Example 5. Show that $\frac{5}{6}$ and $\frac{15}{18}$ are equivalent fractions.

Solution: let $\frac{5}{6} = \frac{15}{18}$, then 5 × 18 = 6 × 15 = 90.

This is another method for checking the equivalenc of two fractions.

1.1.4. Rational Numbers

In sub-section 1.1.1. you have revised important ideas about integers. Integers are represented on a number line as shown below in Figure 1.8.

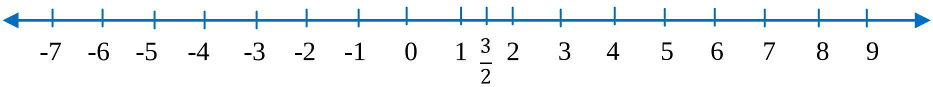


Figure 1.8 Number line

Consider the number $\frac{3}{2}$, it is greater than 1 but less than 2. So it belongs to the interval between 1 and 2 as shown in Figure 1.8. $\frac{3}{2}$ is not a natural number or a whole number and also it is not an integer. It is called **a rational number.**

Using the above discussion, we define the set of rational numbers as follows:

Definition 1.2: Any number that can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$, is called a **rational number.**

Note: i) The set of rational numbers is denoted by $\mathbb{Q}$ such that

$$\mathbb{Q} = \left\{\frac{a}{b} : a, b \in \mathbb{Z} \text{ and } b \neq 0\right\}.$$

ii) Any integer 'a' can be written in the form of $\frac{a}{b}$ where b = 1, it follows that any integer is a rational number.

Example 6. $\frac{1}{7}, \frac{-3}{11}, \frac{1}{5}, \frac{7}{6}, \frac{-8}{9}$ and 11 are rational numbers.

The integer 11 is a rational number since it can be written as $\frac{11}{1}$.

1.1.5. Representing Rational Numbers on a Number Line

Example 7. Sketch a number line and mark the location of each fraction.

a. $\frac{3}{2}$ b. $\frac{3}{4}$ c. $\frac{-5}{2}$ d. $\frac{-3}{2}$ e. $\frac{8}{2}$

Solution: First draw a number line and mark the location of each fraction.

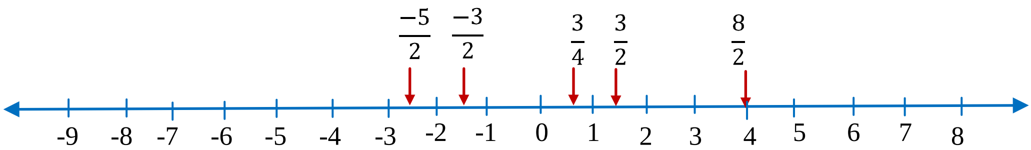


Figure 1.9 Number line

a) The number $\frac{3}{2}$ is located half way between 1 and 2.

b) The number $\frac{3}{4}$ is located between 0 and 1.

c) The number $\frac{-5}{2}$ is located half way between -2 and -3.

d) The number $\frac{-3}{2}$ is located half way between -1 and -2.

e) The number $\frac{8}{2}$ is located at the point labeled 4, $\left(\text{since } \frac{8}{2}=4\right)$.

This description of rational numbers on the number line leads to the following property.

Property of rational numbers

Every rational number corresponds to some unique point on a number line.

In Figure 1.9 above, we will see the following Notes:

i. The new numbers marked on the number line to the left of the zero point and fractions between them are called **Negative rational numbers**. The set of negative rational numbers are denoted by " $\mathbb{Q}^-$".

ii. The new numbers marked on the number line to the right of the zero point and fractions between them are called **positive rational numbers**. The set of positive rational numbers are denoted by " $\mathbb{Q}^+$".

iii. The union of the set of positive rational numbers, set containing zero and the set of negative rational numbers is called the **set of rational numbers**.

Example 8. Calculate

a. $\frac{3}{5}+\frac{7}{10}$ b. $\frac{19}{18}-\frac{13}{9}$ c. $\frac{-2}{7}\times\frac{8}{11}$ d. $\frac{3}{13}\div\frac{2}{5}$

Solution:

a. $\frac{3}{5}+\frac{7}{10}=\frac{3\times 2}{5\times 2}+\frac{7}{10}=\frac{6}{10}+\frac{7}{10}=\frac{6+7}{10}=\frac{13}{10}.$

b. $\frac{19}{18}-\frac{13}{9}=\frac{19}{18}-\frac{13\times 2}{9\times 2}=\frac{19}{18}-\frac{26}{18}=\frac{19-26}{18}=\frac{-7}{18}.$

c. $\frac{-2}{7}\times\frac{8}{11}=\frac{-2\times 8}{7\times 11}=\frac{-16}{77}.$

d. $\frac{3}{13}\div\frac{2}{5}=\frac{3}{13}\times\frac{5}{2}=\frac{15}{26}.$

From this example you can easily see that the sum, difference, product and quotient of two rational numbers are also rational numbers.

In grade 6 mathematics you had learnt how to convert a given terminating decimal to a fraction.

? Do you remember how you did that?

Look at the following examples carefully.

Example 9. Convert each decimal given below to a fraction.

a. 0.25 b. 2.4 c. 1.28

Solution:

a. $0.25 = 0.25 \times \frac{100}{100} = \frac{0.25\times100}{100} = \frac{25}{100} = \frac{1\times25}{4\times25} = \frac{1}{4}$. Thus $0.25 = \frac{1}{4}$.

b. $2.4 = 2.4 \times \frac{10}{10} = \frac{2.4\times10}{10} = \frac{24}{10} = \frac{12\times2}{5\times2} = \frac{12}{5}$. Thus $2.4 = \frac{12}{5}$.

c. $1.28 = 1.28 \times \frac{100}{100} = \frac{1.28\times100}{100} = \frac{128}{100} = \frac{32\times4}{25\times4} = \frac{32}{25}$. Thus $1.28 = \frac{32}{25}$.

As you can see from example 9 above, terminating decimals can be expressed as fractions. So we can say that terminating decimals are rational numbers.

Exercise 1A

1. Compute the following ploblems in $\mathbb{Q}$.

   a. $270 + 80$ d. $23.9 + 28.9$ g. $\left(\frac{1}{4} + \frac{1}{2}\right) \div \frac{3}{2}$

   b. $320 - 90$ e. $49.72 - 58.87$ h. $\left(\frac{1}{5} - \frac{1}{2}\right) \div \frac{3}{5}$

   c. $2.7 + 2.8$ f. $\frac{2}{5} - \frac{3}{7}$ i. $\left(\frac{2}{5} \div \frac{5}{6}\right) \times \frac{7}{8}$

2. Are the following pairs of fractions equivalent? Give the reasons to your answer.

   a. $\frac{5}{9}$ and $\frac{20}{36}$ b. $\frac{1}{7}$ and $\frac{9}{63}$ c. $\frac{625}{25}$ and 25 d. $\frac{90}{121}$ and $\frac{6}{81}$

3. Find at least four equivalent fractions for each fraction.

   a. $\frac{8}{18}$ b. $\frac{30}{36}$

4. All of the following expressions represent rational numbers. Rewrite each of them in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

   a. $6\frac{1}{2}$ c. -23 e. -0.83 g. $4\frac{7}{8}$

   b. $1\frac{7}{8}$ d. -4.33 f. 8763.2 h. $2\frac{7}{100}$

5. Draw a number line and represent the following rational numbers on a number line.

   a. 5   c. $\frac{-5}{2}$ e. -8   g. $\frac{16}{8}$ i. $\frac{283}{283}$

   b. $3\frac{1}{5}$ d. $\frac{1}{2}$ f. $2\frac{5}{6}$ h. $\frac{25}{6}$

Challenge Problems

6. There ae 28 people on a martial arts course. 13 are female and 15 are male. What fraction of the people are:

   a. Male   b. Female

7. Represent the following fact by using a numeral and + and – signs.

   a. A loss of Birr 100.   d. Five minutes Late.

   b. A rise of 10^{o}C temperature.   e. 28^{o} below zero.

   c. A walk of 5km forward.   f. 46^{o}C above zero.

1.1.6. Relationship Among $\mathbb{W}$, $\mathbb{Z}$ and $\mathbb{Q}$

Group work 1.2

Discuss with your friends/Group

The Venn diagram below shows the relationships between the set of Natural numbers, Whole numbers, Integers and Rational numbers.

1. **List three numbers that are rationals but not integers.**
2. **List three numbers that are integers but not whole numbers.**
3. **List three numbers that are integers but not natural numbers.**
4. **What relations have you observed between the sets of natural numbers, whole numbers, integers and rational numbers.**

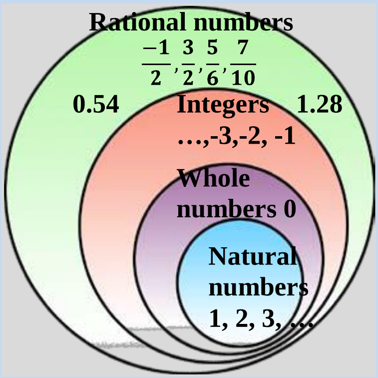


Figure 1.10

5. **What is the intersection of the set of integers and rational numbers?**
6. **What is the union of the set of whole numbers and the set of rational numbers.**

In Figure 1.10 above, we will see the following facts listed as follows:

Note: The set of whole numbers includes the natural numbers. Therefore, every natural number is also a whole number.

- ✓ The set of integers includes the set of whole numbers. Therefore, every whole number is also an integer.
- ✓ The set of rational numbers includes the set of integers. Therefore, every integers is also a rational number.
- ✓ The relationship among the elements of natural numbers, whole numbers, integers and rational numbers is shown in Figure 1.10 above.
- ✓ The set of whole numbers is a subset of the set of integers and the set of integers is the subset of the set of rational numbers or $\mathbf{W} \subseteq \mathbb{Z} \subseteq \mathbb{Q}$.

1.1.7. Opposite of a Rational Number

Activity1.4

Discuss with your teacher orally

1. **Find the opposite of each integer given below.**

   a. 70 b. -23 c. -170 d. 0

2. **Can you give the opposite of each rational number given below?**

   a. $-\frac{1}{3}$ c. $\frac{1}{20}$ e. 4.5 g. $3\frac{2}{5}$

   b. $\frac{45}{2}$ d. -4.5 f. -0.6

Each point on the number line has another point opposite to it with respect to the point corresponding to zero. The numbers corresponding to these two points are called **opposites** of each other. A number and its opposite are always found at the same distance from zero as shown in Figure 1.11 below.

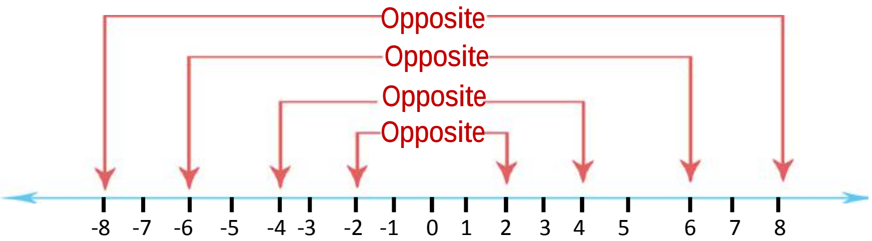


Figure 1.11 opposite numbers

From the above discussion, we define opposite of rational numbers as follows:

Definition 1.3: Two rational numbers whose corresponding points on the number line that are found at the same distance from the origin but on opposite sides of the origin are called opposite numbers.

Note: i) As a special case, we will agree that 0 is its own opposite.

ii) In general the opposite of a rational number 'a' is denoted by '-a'. Thus the opposite of $-a$ is $-(-a) = a$, $a \neq 0$.

iii) Every rational number has an opposite.

Example 10. Find the opposite of each integer given below.

a. -10 b. -15 c. 60 d. 25

Solution:

a. -10 is the opposite of 10

b. -15 is the opposite of 15

c. 60 is the opposite of -60

d. 25 is the opposite of -25

Note: On the number line the points corresponding to the integers in each pair above are found on opposite sides but the same distance from the origin.

Example 11. Find the opposite of 8 with the help of a number line.

Solution: First draw a number line and start from the origin move 8 units to the positive direction, next start from the origin and move 8 units to the left.

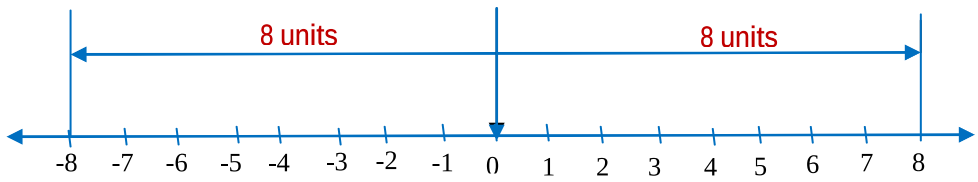


Figure 1.12 number line

Hence the opposite of 8 is -8.

Note: ***Properties of Opposites***

i) If a is positive, then its opposite – a is negative.
ii) Number zero is the opposite of itself.
iii) If a is negative, then its opposite +a is positive.

Example 12. Find the opposite of each rational number.

a. If a = 65, then –a = -65, is the opposite of a.

b. If a = $\frac{27}{91}$, then –a = $\frac{-27}{91}$, is the opposite of a.

c. If a = -75, then –a = -(-75) = 75, is the opposite of a.

d. If a = $\frac{-39}{71}$, then –a = $-\left(\frac{-39}{71}\right) = \frac{39}{71}$, the opposite of a.

Note: $\frac{-12}{23} = \frac{12}{-23} = -\frac{12}{23}$ are different representations of the same number that is the opposite of $\frac{12}{23}$.

Exercise 1B

1. Which of the following statements are true and which are false?

a. $\frac{5}{2} \in \mathbb{W}$	d. $\frac{-5}{2} \in \mathbb{Q}^+$	g. $-0.67 \in \mathbb{Q}^-$	j. $-5 \in \mathbb{Z}^-$
b. $-70 \in \mathbb{W}$	e. $0 \in \mathbb{Q}^-$	h. $-3.25 \in \mathbb{N}$	k. $0.668 \in \mathbb{Q}$
c. $0 \in \mathbb{W}$	f. $0.5 \in \mathbb{Q}$	i. $0.2 \in \mathbb{Z}^+$	

2. Which of the following statements are true and which are false?

a. $\mathbb{N} \subseteq \mathbb{W}$	d. $\mathbb{Z}^- \subseteq \mathbb{Z}$	g. $\mathbb{Z} \subseteq W$	j. $\mathbb{Q}^- \subseteq \mathbb{Q}^+$
b. $\mathbb{N} \subseteq \mathbb{Z}^+$	e. $\mathbb{W} \subseteq \mathbb{Z}^+$	h. $\mathbb{Z} \subseteq \mathbb{Q}$	k. $\mathbb{Q}^- \subseteq \mathbb{Q}$
c. $\mathbb{Z}^+ \subseteq \mathbb{Q}^+$	f. $\mathbb{W} \subseteq \mathbb{Z}^-$	i. $\mathbb{Q}^+ \subseteq \mathbb{Q}$	

3. Find the opposite of each rational numbers.

a. 0.823	d. $3\frac{3}{39}$	g. 8.797
b. -26.72	e. $\frac{-8}{50}$	h. $20\frac{5}{80}$
c. -24.278	f. $\frac{0}{10,000}$	i. $36\frac{70}{80}$

4. Determine the value of x.
   a. $x = -(-28)$ c. $-x = 0$
   b. $-x = 3\frac{5}{9}$ d. $-x = -(-70)$
5. Write a number, the opposite of which is
   a. Positive b. Negative c. neither positive nor negative

Challenge Problem

6. Use your own Venn diagrams to show all the possible relationhips among $\mathbb{W}$, $\mathbb{Z}$ and $\mathbb{Q}$.

1.1.8. The Absolute Value of a Rational Number

Activity 1.5

Write each of the following with out the absolute value sign:

a) $|8|$ **b)** $|-8|$ **c)** $|0|$ **d)** $\left|\frac{1}{2}\right|$

The **absolute value** of a rational number can be defined as the distance from zero on the number line. The symbol for the absolute value of a number 'x' is $|x|$. Since points coprreponding to 12 and -12 are at the same distance from the points corresponding to 0, we have, $|-12|=12$ and $|12|=12$.

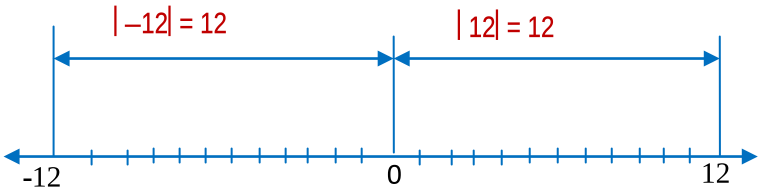


Figure 1.13 Number line

From the above discussion we have the following true or valid statements.

a) If x is a positive number, then $|x| = x$.

Example 13. a) $|5| = 5$ since the absolute value of a positive rational number is the number itself.

b) $|0| = 0$ since the absolute value of zero is zero.

c) If x is a negative number, then $|-x| = -(-x) = x$.

Example 14: $|-5| = 5$, since the absolute value of a negative rational number is the opposite of the number.

Note: $|\mathbf{12}| = 12$ is read as "the absolute value of (positive) twelve is twelve". $|-\mathbf{12}| = 12$ is read as "the absolute value of (negative) twelve is twelve".

Definition 1.4: The absolute value of a rational number 'x' is denoted by the symbol | x | and defined by:

$$|\mathbf{x}| = \begin{cases} \mathbf{x} \text{ if } \mathbf{x} > 0 \\ \mathbf{0} \quad \text{if } \mathbf{x} = 0 \\ -\mathbf{x} \text{ if } \mathbf{x} < 0 \end{cases}$$

Example 15. Simplifying each of the following absolute value expression.

a) $|7-2|$ b) $|5-10|$ c) $|0-\pi|$

Solution:

a) since 7-2 = 5 and 5> 0, we have $|7-2| = |5| = 5$.

b) Since 5 – 10 = -5 and – 5 < 0, we have $|5-10| = |-5| = -(-5) = 5$.

c) Since $0-\pi = -\pi$ and $-\pi < 0$, we have $|0-\pi| = |-\pi| = -(-\pi) = \pi$.

Equations Involving Absolute Value

Geometrically the expression $|x| = 3$ means that the point with coordinate x is 3 units form 0 on the number line. Obviously the number line contains two points that are 3 units from the origin. One to the right of the origin and the other to the left. Thus $|x| = 3$ has two solution x = 3 and x = -3.

Note: The solution of the equation $|\mathbf{x}| = a$

For any rational number a, the equation $|x| = a$ has

i. two solutions $x = a$ and $x = -a$ if $a > 0$.

ii. one solution, $x = 0$ if $a = 0$ and

iii. no solution, if $a < 0$.

Example 16. Solve the following absolute value equations.

a) $|x| = 5$ b) $|x| = -70$

Solution:

a) $|x| = 5$

If $|x| = 5$, then $x = 5$ or -5

Therefore, the Solution set or S.S= {-5, 5}.

b) $|x| = -70$

The absolute value of a number can not be negative, hence the solution set is empty set or S.S={ }.

Exercise 1C

1. Copy and complete table 1.1 below.

x	8	$\frac{-1}{2}$	$2\frac{3}{2}$	$-5\frac{6}{7}$	-9					$\frac{9}{2}$	2.6	-3.7
$\lvert x \rvert$						0	5.6	0.92	11			

2. Find all rational numbers whose absolute values are given below.

a) $8\frac{3}{5}$ b) 3.5 c) $\frac{2}{5}$ d) $4\frac{1}{6}$ e) 3.8 f) 0

3. Evaluate each of the following expression.

a) $|-7|+|31-11|$

b) $|-18|-|-7|+|5|$

c) $|9+(-9)|$

d) $|4-5|$

e) $|-3+10|$

f) $|3+30|$

g) $|4|+|-10|-|-3|$

h) $|-3|+|25-21|$

4. Evaluate each expression.

a) $-6x+2|x-3|$, when $x = -3$

b) $|m|-m+3$, when $m = \frac{1}{2}$

c) $|x|+|y|$ when $x = -3$ and $y = -1$

d) $|y|-|x|$ When $y = -7$ and $x = 3$

e) $(|9-y|)\times(-1)$ when $y = -5$

f) $-2|x-7|$, when $x = -3$

5. Solve the following absolute value equations.

a) $|x| = 2\frac{3}{5}$

b) $|x| = 2.35$

c) $1-2|x+2|=6$

d) $2|x-5|+7=14$

e) $|4x| = 32$

f) $|x-4|=7$

Challenge Problems

6. Solve the following absolute value equations.

a) $|8-12x| = 3\frac{2}{5}$

b) $-3|2x+10|+2=27$

1.2 Comparing and Ordering Rational Numbers

Group work 1.3

1. **Is there any integer between n and n+1, where n $\in \mathbb{N}$?**
2. **Is there any whole number between n and n + 1 where n $\in \mathbb{N}$?**
3. **Arrange the following integers in ascending order:**
   -70 , - 10, 0, 52 , 43 , 65 , 34
4. **Arrange the following integers in a descending order:**
   -5 , - 10 , 0, 16, 70, 100
5. **Name all integers which lie between:**
   a) -5 and 2
   b) -2 and 10
   c) 0 and 3
   d) 2 and 5
6. **Insert >, = or < to express the corresponding relationship between the following pairs of intigers.**
   a) 0 ______ 500
   b) 3 ______ - 100
   c) $\frac{150}{3}$ ______ $\frac{200}{5}$
   d) $\frac{50}{2}$ _____ $\frac{-16}{4}$
   e) 50 _____1023
   f) -120 _____-120

The concept **"Less than"** for rational numbers is similar to that of integers. Recall that for integers, the smaller of two numbers was to the left of the larger on the number line. As shown in Figure 1.14 below $\frac{-6}{5}$ lies to the left of $\frac{-1}{5}$ and $\frac{1}{5}$ lies to the left of $\frac{6}{5}$. Therefore; $\frac{-6}{5} < \frac{-1}{5}$ and $\frac{1}{5} < \frac{6}{5}$.

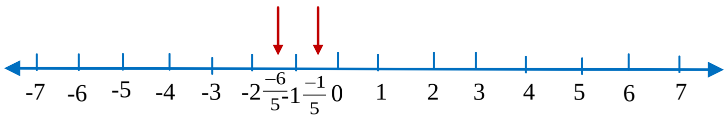


Figure 1.14 Number line

All of these fractions have the same denominator, 5 it follows that
-6 < - 1 and 1 < 6.

Example 17. Consider the number line given in Figure 1.15 below.

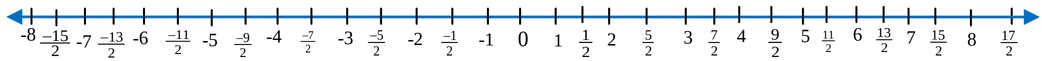


Figure 1.15 Number line

As shown in the above number line -5 is to the left of -2; $\frac{-9}{2}$ is to the left of $\frac{-7}{2}$; $\frac{-5}{2}$ is to the left of $\frac{-1}{2}$; $\frac{1}{2}$ is to the left of 2; and $\frac{7}{2}$ is to the left of 5.

There fore $-5 < -2$, $\frac{-9}{2} < \frac{-7}{2}$, $\frac{-5}{2} < \frac{-1}{2}$, $\frac{1}{2} < 2$ and $\frac{7}{2} < 5$.

Note: For any two different rational numbers whose corresponding points are marked on the number line, the one located to the left is smaller.

Example 18. Compare the following pair of numbers.

a) – 5 and 5

b) -2.5 and -3.5

c) $2\frac{5}{7}$ and $3\frac{2}{7}$

d) $3\frac{2}{3}$ and $2\frac{5}{9}$

Solution:

a) $-5 < 5$

b) $-2.5 > -3.5$

c) $2\frac{5}{7} \leq 3\frac{2}{7}$

d) $3\frac{2}{3} \geq 2\frac{5}{9}$

Example 19. Draw a number line and represent the following equivalent rational numbers on a number line.

a) $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}, \frac{8}{16}, \frac{16}{32}$

b) $\frac{5}{7}, \frac{30}{42}, \frac{210}{294}$

Solution: a) All $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}, \frac{8}{16}$ and $\frac{16}{32}$ have the same point $\frac{1}{2}$ therefore,

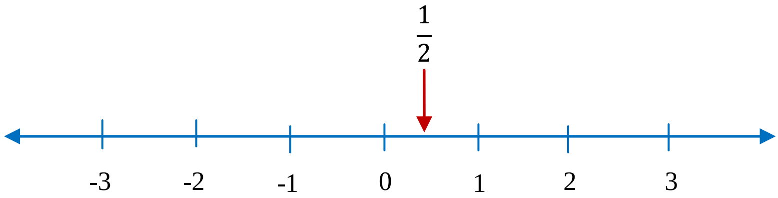


Figure 1.16 Number line

b) All point $\frac{5}{7}, \frac{30}{42}$ and $\frac{210}{294}$ have the same point $\frac{5}{7}$ that corresponding to $\frac{5}{7}$ therefore

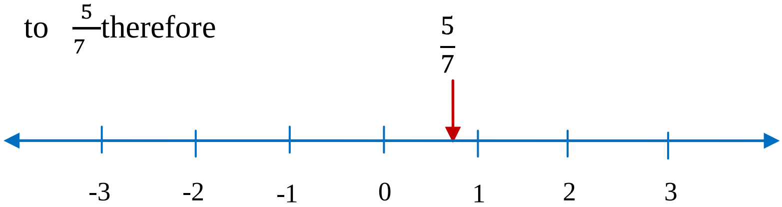


Figure 1.17 Number line

From the above fact, it follows that:

- ✓ Every positive rational number is greater than Zero.
- ✓ Every negative rational number is less than Zero.
- ✓ Every positive rational number is greater than every negative rational number.
- ✓ Among two negative rational numbers, the one with the largest absolute value is smaller than the other.

  For example, $-76 < -7$ because $|-76| > |-7|$.

Note: In the system of rational numbers, there are many rational numbers between any two rational numbers.

Example 20. $-1.2, -1.25, -1.5, -1.6$ and $\frac{-7}{4}$ are between -2 and -1.

Definition 1.5: (ordering similar fraction)

Let $\frac{a}{c}$ and $\frac{b}{c}$ be any fractions with $c > 0$, then $\frac{a}{c} < \frac{b}{c}$ if and only if $a < b$.

Example 21. a) $\frac{-26}{7} < \frac{-18}{7}$ because $-26 < -18$.   c) $1\frac{2}{3} < 2\frac{5}{3}$ because $5 < 11$.

b) $\frac{39}{17} < \frac{45}{17}$ because $39 < 45$.

✓ To test whether $\frac{2}{3}$ is less than $\frac{3}{4}$, we change $\frac{2}{3}$ and $\frac{3}{4}$ to equivalent fractions (fractions with the same denominator).

$\frac{2}{3} = \frac{2\times4}{3\times4} = \frac{8}{12}$ and $\frac{3}{4} = \frac{3\times3}{4\times3} = \frac{9}{12}$, then by comparing $\frac{8}{12}$ and $\frac{9}{12}$, i.e, $\frac{8}{12} < \frac{9}{12}$

Therefore, $\frac{2}{3} < \frac{3}{4}$ (because $2\times4 = 8 < 9 = 3\times3$).

This example suggests the following definition:

Definition 1.6: ordering dissimilar fractions

If $\frac{p}{q}$ and $\frac{r}{s}$ are rational numbers expressed with positive denominators, then $\frac{p}{q} < \frac{r}{s}$ if and only if $ps < qr$.

Example 22. a) $\frac{6}{9} < \frac{10}{8}$ because $6 \times 8 < 9 \times 10$ that is $48 < 90$.

b) $\frac{-8}{10} < \frac{2}{8}$ because $-8 \times 8 < 10 \times 2$ that is $-64 < 20$.

c) $\frac{4}{3} > \frac{6}{7}$ because $4 \times 7 > 3 \times 6$ that is $28 > 18$.

From the above fact, it follows that:

i. Relations Among Numbers:

If a and b represent rational numbers, then one and only one of these relations can be true:

a is equal to b or a is less than b or a is greater than b or

$a = b$ or $a < b$ or $a > b$.

ii. If $a \neq b$, then $a < b$ or $a > b$.

Exercise 1D

1. Which of the following statements are true and which are false.

a) $-3\frac{1}{2} < |-2.8|$

b) $\left|\frac{-10}{7}\right| = \left|\frac{-2}{7}\right|$

c) $|0.2| = \left|\frac{1}{5}\right|$

d) $\frac{3}{4} > \frac{1}{4}$

e) $|-8.6| > 8.6$

f) $\frac{5}{8} < \frac{6}{5}$

g) $\left|2\frac{3}{5}\right| > \left|\frac{13}{5}\right|$

h) $\left|\frac{-5}{2}\right| < \frac{5}{2}$

i) $\left|\frac{2}{3}\right| + \left|\frac{-1}{3}\right| = 1$

j) $\left|\frac{-7}{3}\right| < \frac{4}{3}$

k) $\left|\frac{2}{3}\right| > \left|\frac{-3}{4}\right|$

2. Insert (>, = or <) to express the corresponding relationship between the following pair of numbers.

a) $\frac{8}{30}$ ____ $\frac{6}{30}$

b) $\frac{-24}{18}$ ____ $\frac{-8}{6}$

c) $\frac{15}{4}$ ____ $\frac{8}{3}$

d) $3\frac{2}{3}$ ____ $1\frac{2}{5}$

e) $|-70|$ ____ 70

f) -0.5 ____ $\frac{-1}{2}$

g) -0.92 ____ -0.89

h) $\frac{-3}{4}$ ____ -0.75

i) $|700|$ ____ -700

j) $27 - 3$ ____ 6

k) $20 + 6$ ____ $50 + 30$

3. From each pair of numbers below which number is to the right of the other?

a) 25, 7

b) $\frac{3}{5}, \frac{6}{8}$

c) $3\frac{2}{3}, 2\frac{1}{7}$

d) $\frac{5}{8}, \frac{-3}{5}$

e) $-5, \frac{15}{2}$

f) $\frac{1}{2}, -1.2$

4. Abebe, Almaz and Hailu played Basket balls. The results are shown in table 1.2 .

	First Play	Second play	Final result
Abebe	Loss 5 Basket balls	Won 7 Basket balls	
Almaz	Won 6 Basket balls	Loss 6 Basket balls	0
Hailu	Loss 3 Basket balls	Loss 2 Basket balls	

Complete in table 1.2. Who was the winner? Who was the looser of the competition?

5. The five integers x, y, z, n and m are represented on the number line below.

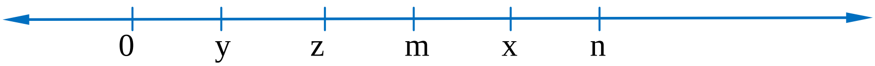


Figure 1.18 Number line

Using < or > fill in the blank spaces.

a) z _______ x   c) n _______ y

b) m _______ x   d) z _______ n

6. a, b, c, d, e, f are natural numbers represented on a number line as follows:

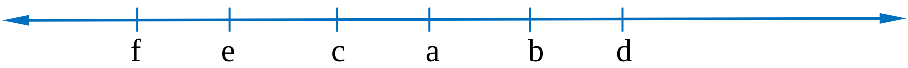


Figure 1.19 Number line

Copy and complete by writing > or <.

a) a_____ b   c) b ____ c   e) d ____ a____ e

b) a ____ c   d) d _____c

7. x is a natural number such that x < 10

a) List the possible values of x.

b) Represent the possible values of x on a number line.

8. Put the numbers in the cloud in order. Start with

a. Descending order.

b. Ascending order.

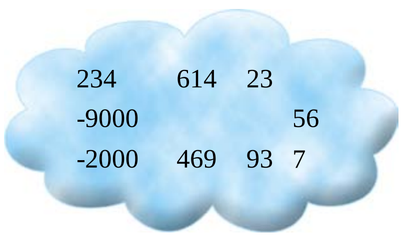


Figure 1.20 numbers in the cloud

1.3. Operation on Rational Numbers

1.3.1. Addition of Rational Numbers

Group work 1.4

1. **Draw a number line to show how to find the sum of each of the following rational numbers.**

   a) 6+5   d) 9+5   g) $\frac{3}{2}+\frac{7}{4}$ i. 7+10

   b) 14+1   e) -10 +2

   c) 5+2+7   f) -14+8   h) $\frac{3}{4}+\frac{6}{5}$

2. **Write each sum as a fraction or mixed number.**

   a) $\frac{4}{5}+\frac{3}{5}$ b) $\frac{2}{9}+\frac{6}{9}$ c) $\frac{5}{3}+\frac{2}{3}$

3. **Give (or approximate) the numbers represented by the letters on the number line below.**

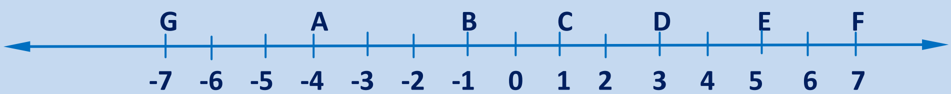


Figure 1.21 number line

4. **Indicate the positions of the points corresponding to the numbers -2, -5, 0 and 3 on a number line.**
5. **Find the following sums. Give your answer in simplest form.**

   a) $\frac{2}{3}+\frac{1}{5}$ c) $\frac{3}{5}+\frac{5}{2}$ e) $\frac{1}{4}+\frac{5}{4}+\frac{3}{8}$

   b) $\frac{1}{5}+\frac{7}{9}$ d) $\frac{1}{7}+\frac{2}{3}+\frac{5}{2}$ f) $\frac{5}{1}+\frac{1}{6}$

1.3.1.1 The Sum of Two Numbers of the Same Sign

You can picture the addition of two numbers on a number line by three arrows: an arrow for each addend and an arrow for the sum.

- ✓ The first arrow starts at the origin.
- ✓ The second arrow starts where the first arrow ends.
- ✓ The arrow for the sum starts where the first arrow starts and ends where the second arrow ends.

The arrows in the number line below show an addition in which both addends are positive: $2+3 = 5$.

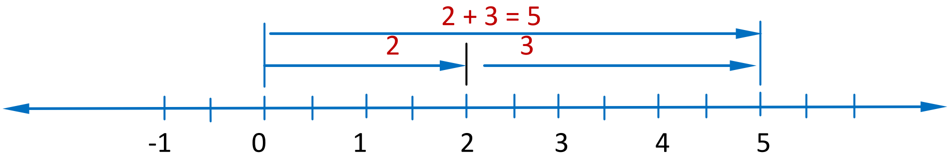


Figure 1.22 Number line

Since the addends and their sum are positive, all three of the arrows are directed to the right. This suggests the following properties.

Note: The sum of two positive numbers is a positive number.

Example 23. Find the sum of 2.8 + 4.6.

Solution:

The length of the arrow for the sum is 2.8 + 4.6 or 7.4 Since both addends are positive, the sum is positive. Hence 2.8 + 4.6 = 7.4.

The number line below shows an addition in which both addends are negative: $-3 + (-1) = -4$.

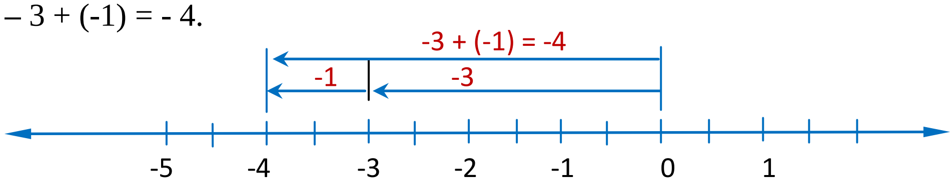


Figure 1.23 Number line

Since the addends and their sum are negative, all the three arrows are directed to the left.

Note: The sum of two negative numbers is a negative number.

Example 24. Find the sum of -2.8 and -3.5.

Solution:

The length of the arrow for the sum is 2.8 + 3.5 or 6.3. Since each of the addends is negative, the sum is negative. Therefore -2.8+(-3.5)= -6.3.

1.3.1.2 The sum of Two Numbers of Opposite Sign

i) When the addend with the longer arrow is positive, the sum is positive.

ii) When the addend with the longer arrow is negative, the sum is negative.

iii) When the addends have arrows of equal length the sum is 0.

Example 25. Draw a number line and find the sum by arrow addition.

a) 5.5 + (-3.2) b) -5+ 3 c) 3 + (-3)

Solution:

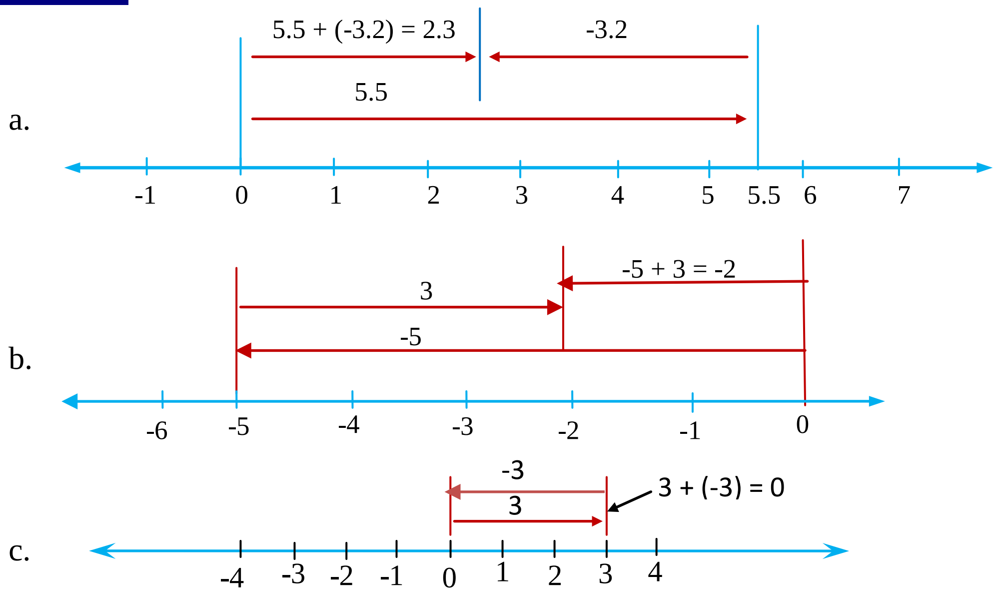


Figure 1.24 number lines

The brackets for the second addend if it is negative are used for clear distinction between the positive operation sign ("+") and the sign for the negative rational number (" –") for negative.

If the negative rational number is placed as the first addend, its sign cannot be mixed up with operation sign for subtraction.

Exercise 1E

1. Write the sum.

a) $8.2 + (-3.2)$
b) $28+(-36)$
c) $7.4 + (-2.8)$
d) $-248+236$
e) $42 + (-54)$
f) $0+ (-9.6)$
g) $8+ (-96)$
h) $-10.25 + 6.54$
i) $6\frac{1}{2} + 5\frac{3}{2} + \left(-2\frac{1}{2}\right)$
j) $3\frac{1}{5} + \left(\frac{-7}{8}\right) + 3\frac{6}{5}$

2. a) Find two rational numbers whose sum is – 10, 0 and 15.

b) Find four rational numbers whose sum is 30, - 28, and 70.

3. Simplify each of the following addition.

a) $x + (-x) + r$
b) $2m + 3m + (-6m)$
c) $2m + (-2m) + 4m$
d) $20 + (-6x) + 6x + (-20)$
e) $-2d + (-70a) + 3d$
f) $6x + \left(\frac{-12x}{2}\right) + \left(\frac{-24x}{2}\right)$

4. On a number line add the following by using arrows.

a) $5+ (-2)$
b) $-6+4$
c) $2+ (-3)$
d) $4+ (-4)$
e) $3 + (-5)$

In each of Exercise 5 and 6, state the first addend, the second addend and the sum by writing the resulting equation.

5.

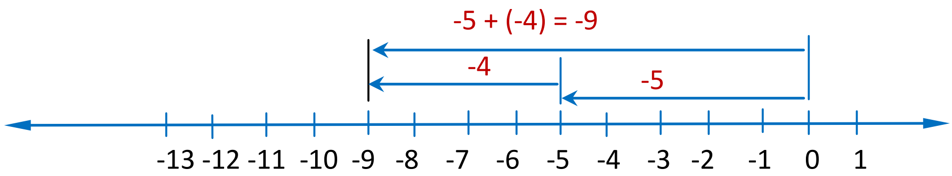


Figure 1.25 number line

6.

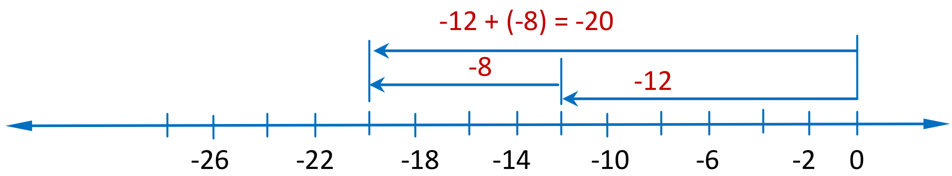


Figure 1.26 number line

7. In Figure 1.27 below, state the value of the missing addend (d).

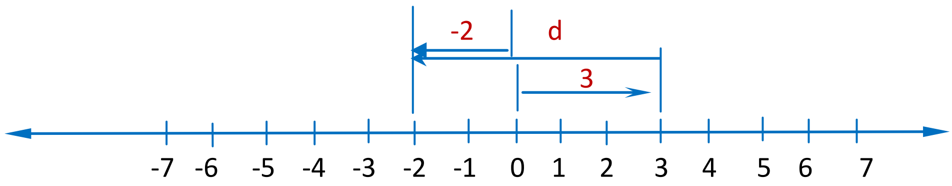


Figure 1.27 number line

8. Atnafu was playing a two round game in which he could gain or lose points. During the first round he lost 28 points. During the second round he gained 10 points. What was his net score at the end of the game?

Challenge Problems

9. Find the two natural numbers whose sum is 30, where one of the numbers is five times the other.
10. If 4x + 8x + 12x + 16x = 5 + 10 + 30 + 40, then what is the value of x?

1.3.1.3 Rules for the Addition of Rational Numbers

✓ Now you are going to discover some efficient rules for adding any two rational numbers. Since you know how to add any positive rational numbers and you also know result you get when you add zero to any rational number. You will concentrate now on two rules:

Rule 1: To find the sum of rational numbers where both are negative:

i) Decide (put) the sign first.

ii) Take the sum of the absolute values of the addend.

iii) Put the sign infront of the sum.

Example 26. Perform the following addition:

a) -5 + (-7)

b) $\frac{-3}{2}+\left(\frac{-3}{8}\right)$

Solution: a) -5 + (- 7)

i) sing (-)

ii) Absolute value: 5+ 7 = 12

Hence: – 5 + (-7) = - 12

b) $\frac{-3}{2}+\left(\frac{-3}{8}\right)$

i) sing (-)

ii) Absolute value $\frac{3}{2}+\frac{3}{8}=\frac{15}{8}$

Hence $\frac{-3}{2}+\left(\frac{-3}{8}\right)=\frac{-15}{8}$

Rule 2: To find the sum when the signs of the addends are different (or adding a negative and positive) rational numbers as follows:

i) Take the sign of the addend with the greater absolute value.

ii) Take the absolute values of both numbers and subtract the addend with smaller absolute value from the addend with greater absolute value.

iii) Put the sign infront of the difference.

Example 27. Perform the following addition:

a) – 5+ 9

b) $\left(\frac{-11}{2}\right)+\left(\frac{5}{2}\right)$

Solution: a) – 5 + 9

i) Sign (+) --- due to $|9|>|-5|$

ii) Difference of absolute values: 9- 5 = 4

Hence – 5 + 9 = 4

b) i) sign (-) --- due to $\left(\frac{-11}{2}\right)>\left(\frac{5}{2}\right)$

ii) Difference of absolute value:

$$\frac{11}{2}-\frac{5}{2}=\frac{-6}{2}=-3$$

Hence: $\frac{-11}{2}+\frac{5}{2}=\frac{-11+5}{2}=\frac{-6}{2}=-3$

Exercise 1F

1. Write the Sum.

a) $-24+|-7|$

b) $|98|+|49|$

c) $\frac{-5}{6}+\left(\frac{-3}{13}\right)$

d) $26\ \frac{1}{5}+(-0.09)$

e) $\frac{-3}{4}+\frac{5}{8}$

f) 125 + (-75)

2. Solve for the value of x and y.

a) $13x + 10 = 60$

b) $3x - 7(2x - 13) = 3(-2x + 9)$

c) $8 + y = 9$

d) $\frac{x}{8} + \frac{x}{6} = 2$

e) $-628 + 327 = y$

f) $3x + y = 10$ when $y = 2$

3. Evaluate each expression for the given values of x and y.

a) $18 + \frac{x}{2}$ for $x = -8, 18\frac{2}{3}$

b) $y + \left(\frac{-5}{8}\right)$ for $y = \frac{4}{3}, -2.6$

c) $x + \left(\frac{-9}{2}\right)$ for $x = -3, 0.25$

1.3.1.4 Properties of Addition of Rational Numbers

The following properties of addition hold true for any rational numbers.
For any rational numbers a, b and c

a) **Commutative property for addition:** $a + b = b + a$

Example: $5 + 10 = 10 + 5$

$$\frac{1}{7} + \frac{9}{8} = \frac{9}{8} + \frac{1}{7}$$

b) **Associative property for Addition:** $a + (b + c) = (a + b) + c$

Example: $3 + (11 + 5) = (3 + 11) + 5$

$$\frac{3}{5} + \left(\frac{2}{5} + \frac{6}{5}\right) = \left(\frac{3}{5} + \frac{2}{5}\right) + \frac{6}{5}$$

c) **Properties of 0**

$a + 0 = a$

Example $30 + 0 = 30$

$$\frac{3}{5} + 0 = \frac{3}{5}$$

d) **Property of opposites:** $a + (-a) = 0$

Example 28. Use the associative and commutative properties of addition to simplify these additions.

a) $53 + 28 + 47$

b) $576 + 637 + 424 + 863$

Solution:

a) 53 + 28 + 47

= (53 + 28) + 47 ---- Associative property

= (28 + 53) + 47 ---- Commutative property

= 28+ (53 + 47) ----- Associative property

= 28 + 100

= 128 --- Addition Operation

b) 576 + 637 + 424 + 863

= 576 + (637 + 424) + 863 ----- Associative property

= 576 + (424 + 637) + 863 ---- Commutative property

= (576 + 424) + (637 +863) --- Associative property

= 1000 + 1500 = 2500 ---- Addition operation

Exercise 1G

1. Copy and complete the following table 1.3 below:

a	b	c	a+ b	b +a	(a + b) + c	a+ (b+c)
6	**-8**	**14**				
-2.3	**-5.6**	**9.6**				
¾	**-5/7**	**-2.5**				

What do you understand from this table?

2. Use the commutative and associative properties to simplify the steps of addition of the following. Mention the property you used in each step.

a) 34 + 48+ 66

b) 218 + 125+782+375

c) 59+42+41+36

d) 572+324+176+447+428+253

e) 3.7+5.8+0.8+0.9

f) 3.9 + 0.8+0.66+3 $\frac{5}{2}$

1.3.2 Subtraction of Rational Numbers

Activity 1.6

1. Find the differences -5$\frac{1}{3}$ − 12.

2. Find each of the following differences.

a) $1\frac{3}{4} - \left(-2\frac{1}{2}\right)$ **b)** $\frac{0}{4} - \left(\frac{-17}{4}\right)$ **c)** $1\frac{2}{7} - \left(-3\frac{5}{7}\right)$ **d)** $-2\frac{1}{2} - \left(12\frac{1}{16}\right)$

✓ Under this sub topic you will see that subtraction of any rational numbers can be explained as the inverse of addition. You may define subtraction as follows:

Subtraction

- ✓ For any numbers a, b and c , a – b = c, if and only if c + b = a.
- ✓ c or a – b is the **difference** obtained by subtracting b from a, a- b is read "a **minus** b".
- ✓ The operation of **subtraction** is denoted by "-".

Example 29. Find the given difference:

a) 5 – 12 b) $\frac{-9}{2}-\left(\frac{-13}{4}\right)$

Solution:

a) Let 5 – 12 = y, then the value of " y" has to satisfy y + 12 = 5
Therefore, y = -7 because – 7 + 12 = 5.

b) Let $\frac{-9}{2}-\left(\frac{-13}{4}\right)=x$, then the value of "x" has to satisfy $x+\left(\frac{-13}{4}\right)=\frac{-9}{2}$

Therefore, $x=\frac{-5}{4}$ because $\frac{-5}{4}+\left(\frac{-13}{4}\right)=\frac{-9}{2}$.

Based on the above information, you can formulate the following property for subtraction of rational numbers.

Property:

For any numbers a and b, a – b = a+ (- b)

Subtract **add the opposite**

Note: i) the difference of two rational numbers is always a rational number.

ii) addition and subtraction are inverse operations of each other in rational numbers.

Example 30. Find the difference by first expressing it as a sum

a) – 7 – (- 6)   b) 28 – 7

Solution: a) -7 - (-6) = -7 + (-(-6)) = -7 + 6 = -1

b) 28 – 7 = 28 + (-7) = 21 or 28- 7 = 21----With out using the rule.

Exercise 1H

1. Find each of the following differences.

a) $18\frac{9}{10}-\left(\frac{-3}{4}\right)$

b) $-5\frac{1}{3}-12\frac{1}{6}$

c) – 0. 5 – (-0.2)

d) $-82.5-|-82.5|$

e) $|10|-6.5$

f) $8-|-6|$

g)`$12-|-7|$

h) $|15|-2.4$

i) $\left|\frac{-3}{4}\right|-\left(\frac{-7}{4}\right)$

2. Copy and complete in table 1.4 below.

a	2	-10	0	14	28	2.8
b	-6	-8	-12	10		
a +b					40	3.8
a-b						

3. Evaluate each expression:

a) 4(1+x) , When x = 2

b) x – (3 –8) +4 When x = 10

c) – x– (7+6) +2 When x = 9

d) 2 – (4- t) when t = 1

e) 12 – (–x) -5, when x = -2

f) -9– (-13) – p When p = -7

4. Show the difference 5- 2 = 5 + (-2) on a number line.

1.3.3. Multiplication of Rational Numbers

Activity 1.7

1. Multiply

a) $\frac{4}{5}\times\frac{2}{7}$

b) $\frac{3}{7}\times\frac{5}{11}$

c) $\frac{-7}{8}\times\frac{-4}{9}$

d) $4\frac{2}{7}\times5\frac{1}{6}$

e) $\frac{-302}{100}\left(\frac{611}{10}\times\left(\frac{-5}{10}\right)\right)$

f) $\frac{-31}{32}\times\left(\frac{-16}{7}\times\frac{2}{62}\right)$

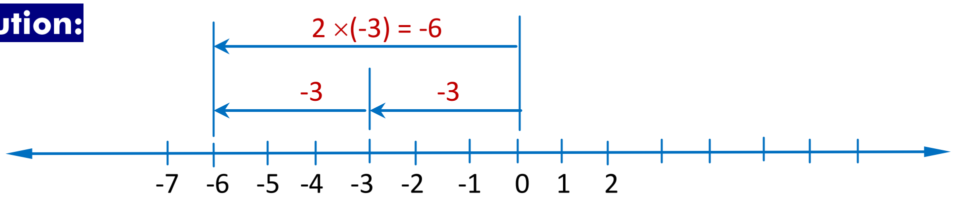
g) $\frac{5}{16}\times\left[\frac{4}{15}\times\left(\frac{-4}{3}\right)\right]$

When you multiply rational numbers use the following fact.

Note: ✓ The product of a negative rational number and a positive rational number is a negative rational number.

Example 31. Find the product $2\times(-3)$ by using a number line.

Solution:



Therefore, $2\times(-3) = -6$

Figure 1.28 Number line

Note: You can find the product of two rational numbers with different signs in three steps:

i. Decide the sign of the product, it is " - ".

ii. Take the product of the absolute value of the numbers.

iii. Put the sign infront of the product.

Example 32. Find the product:

a) -3×5

b) $\frac{3}{8}\times\left(\frac{-5}{7}\right)$

Solution:

a) -3×5

i) Sign (-)

ii) multiply Absolute value or

$$|-3| \times |5| = (3\times 5)$$
$$= 15$$

Hence, $-3\times 5 = -15$

b) $\frac{3}{8}\times\left(\frac{-5}{7}\right)$

i) Sign (-)

ii) Absolutevalue or $=\left|\frac{3}{8}\right|\times\left(\left|\frac{-5}{7}\right|\right)$

$$=\frac{3}{8}\times\left(\frac{5}{7}\right)$$

$$=\frac{-15}{56}$$

Hence, $\frac{3}{8}\times\left(\frac{-5}{7}\right)=\frac{-15}{56}$

Note: The product of two negative rational numbers is a positive rational number.

Example 33. Multiply $-4\frac{2}{7}\times\left(-3\frac{1}{4}\right)$

Solution: First note that the product is positive, then work out with positive numbers only.

$$4\frac{2}{7}\times 3\frac{1}{4}=\frac{30}{7}\times\frac{13}{4}=\frac{195}{14} \text{ or } 13\frac{13}{14}$$

Since the product is positive $-4\frac{2}{7}\times\left(-3\frac{1}{4}\right)=13\frac{13}{14}$

Note: You can find the product of two negative rational numbers in two steps:

i) Decide the sign of the product, it is " +".

ii) take the absolute values of the numbers and multiply them.

Example 34. Find the product:

a) $\frac{-3}{7}\times\left(\frac{-5}{11}\right)$

b) $-4.8\times(-7.8)$

Solution:

a) $\frac{-3}{7}\times\left(\frac{-5}{11}\right)$

i) sign (+)

ii) $\frac{-3}{7}\times\left(\frac{-5}{11}\right)=\left|\frac{-3}{7}\right|\times\left|\frac{-5}{11}\right|$

$=\frac{3}{7}\times\frac{5}{11}$

$=\frac{3\times 5}{7\times 11}$

$=\frac{15}{77}$

Hence $\frac{-3}{7}\times\left(\frac{-5}{11}\right)=\frac{15}{77}$

b. $-4.8\times(-7.8)$

i) sign (+)

ii) $-4.8\times(-7.8)$

$=|-4.8|\times|-7.8|$

$=4.8\times 7.8$

$=37.44$

The following table 1.5 summarizes the facts about product of rational numbers .

The two factors	The product	Example
Both positive	Positive	$3\times5 = 15$
Both negative	Positive	$-3 \times (-5) = 15$
Of opposite sign	Negative	$-3\times 5 = -15$
One or both 0	Zero	$-3\times 0 = 0$

Exercise 1I

1. Express each sum as product.

   a) 0+0+0   c) 5+5+5+5   e) 8+8+8+8

   b) 3+3+3+3   d) 6+6+6+6   f) 50+50+50

2. Express each of the following products as a sum.

   a) 5×1 b) 4×0 c) 5×5 d) 3×3

3. 5 is added to a number. The result is multiplied by 4 and gave the product 32. What was the original number?
4. A number is added to 12. The result is multiplied by 5 and gave the product 105. What was the original number?
5. Adding 6 to a number and then multiplying the result by 7 gives 56. What is the number?
6. Squaring a number and then multiplying the result by 4 gives 1 .What is the number?

Challenge Problems

7. Multiply

   a) $4\frac{3}{4} \times \left[\frac{-16}{15} \times (-3.25)\right]$ b) $\left[4\frac{3}{4} + \left(-1\frac{1}{2}\right)\right] \times \left[-6\frac{1}{8} + \left(-5\frac{3}{8}\right)\right]$

1.3.3.1 Properties of Multiplication of Rational Numbers

Activity 1.8

Which of the following statements are true or false?

a) $4(3+2) = (4\times3) + (4\times2)$

b) $5\frac{3}{6} \times 2\frac{3}{6} = 2\frac{3}{6} \times 5\frac{3}{6}$

c) $2\times(10\times 5) = (2\times 10)\times 5$

d) $2\left(5+3\frac{1}{2}\right) = (2\times 5) + \left(2\times 3\frac{1}{2}\right)$

e) $3\frac{1}{2}\times 0 = 3\frac{1}{2}$

The following properties of multiplication hold true for any rational numbers.

For any rational numbers a, b and c:

1. Commutative property for multiplication: $a \times b = b \times a$

Example: $5\times 70 = 70\times 5$

$$\frac{3}{11}\times\frac{2}{9} = \frac{2}{9}\times\frac{3}{11}$$

2. Associative property for multiplication: $a\times(b\times c) = (a\times b)\times c$

Example: $5\times(7\times 12) = (5\times 7)\times 12$

$$\frac{3}{5}\times\left(\frac{7}{5}\times\frac{8}{5}\right) = \left(\frac{3}{5}\times\frac{7}{5}\right)\times\frac{8}{5}$$

3. Distributive property of multiplication over addition:

$$a\times(b+c) = (a\times b)+(a\times c)$$

Example: $5\times(7+16) = (5\times 7)+(5\times 16)$

$$\frac{2}{3}\times\left(\frac{13}{8}+\frac{1}{9}\right) = \left(\frac{2}{3}\times\frac{13}{8}\right)+\left(\frac{2}{3}\times\frac{1}{9}\right)$$

4. Properties of 0 and 1: $a\times 0 = 0$, $a\times 1 = a$

Examples: $6\times 0 = 0$, $6\times 1 = 6$

Example 35. Use the above property to find the following products.

a) $\frac{3}{7}\times\frac{6}{11}$

b) $2\frac{5}{6}\times 1$

c) 4 (2+3)

d) $(0.67\times 0.8)\times 0$

e) $\frac{-3}{16}\times\left[\frac{2}{15}\times\left(\frac{-4}{3}\right)\right]$

f) $\frac{-11}{32}\times\left(\frac{-8}{7}\times\frac{2}{33}\right)$

Solution:

a) $\frac{3}{7}\times\frac{6}{11} = \frac{3\times 6}{7\times 11} = \frac{18}{77}$

b) $2\frac{5}{6}\times 1 = \frac{17}{6}\times 1 = \frac{17}{6}$

c) $4(2+3) = 4\times 2 + 4\times 3$ ---- Distributive property

$= 8 + 12$

$= 20$

d) (0. 67 × 0.8) × 0 = 0 ……. Property of zero

e) $\frac{-3}{16}\times\left[\frac{2}{15}\times\left(\frac{-4}{3}\right)\right]=\frac{-3}{16}\times\left[\frac{2\times(-4)}{15\times 3}\right]$

$=\frac{-3}{16}\times\left[\frac{-8}{45}\right]$

$=\frac{24}{720}$

f) $\frac{-11}{32}\times\left(\frac{-8}{7}\times\frac{2}{33}\right)=\left[\frac{-11}{32}\times\left(\frac{-8}{7}\right)\right]\times\frac{2}{33}$ ---- Associative property

$=\left[\frac{11\times 8}{32\times 7}\right]\times\frac{2}{33}$

$=\frac{88}{224}\times\frac{2}{33}=\frac{176}{7392}$

Example 36. Simplify each of the following using the properties of rational numbers.

a) 3x + 2 (7x + 5) b) 3x – 7 (2x + 10) c) 2(x + 2y) + 3y

Solution:

a) 3x + 2 (7x+5) = 3x + [(2 × 7x) + (2 × 5)] --- Distributive property
= 3x + [(2+7) x + 2 (5)] --- Associative property of multiplication
= 3x + [14x+10] --- Computation
= [3x + 14x] + 10 ---- Associative property of addition
= [3+ 14] x + 10 --- Distributive property
= 17x +10 --- Computation

b) 3x – 7 (2x+10)
= 3x + (-7)(2x + 10)
= 3x + [-7(2x) + (-7)(10)] ….. Distributive property
= [3x + -14x] + (-70) ….. Associative property
= (3x – 14x) – 70 …. Computation
= (3 – 14)x – 70 ….. Factor out x
= -11x – 70 …. Computation

c) 2 (x + 2y) + 3y
= 2x + 4y + 3y … Distributive property
= 2x + (4y + 3y) … Associative property of addition
= 2x + (4 +3) y … Distributive property
= 2x + 7y … Computation

The following properties can be helpful in simplifying products with three or more factors

1. The product of an even number of negative factors is positive.
2. The product of an odd number of negative factors is negative.
3. A product of rational number with at least one factor 0 is zero.
4. If you multiply a rational number 'a' by -1, then you get the opposite of a, (i.e – a). Therefore, you can write $-1 \times a = -a$.
5. When you multiply a number by a variable, you can omit the multiplication sign and keep the number in front of the variable.

Example 37. Find the Products below:

a) $-4 \times (-7) = 28$

b) $-7 \times 3 = -21$

c) $-7 \times 0 = 0$

d) $-1 \times \frac{5}{2} = \frac{-5}{2}$

e) $a \times 30 = 30a$ (but not $a\,30$)

f) $\frac{5}{2} \times b = \frac{5}{2}b$

Exercise 1J

1. Simplify each of the following using the properties of rational numbers.

a) $-5 + 2(3x + 40)$

b) $5(x + y) + 3(2x + y)$

c) $6(x + 2y) + 2(3x + y)$

d) $4(3+2(x + 5)$

e) $2x^2 + \left[\frac{-4x^2}{2}\right] + 3y^2$

f) $-2x + \left[\frac{-4y}{2}\right] + \frac{x}{2}$

g) $0 + \left[\frac{-2x^2}{2}\right] + 2x^2 + 10$

2. State the properties, in order that are used in these simplifications.

a) $7x + 5x = (7+5)x$ ___________
$= 12x$ _____________

b) $20x + 6x = (20 + 6)x$ ________
$= 26x$_____________

c) $5a + 3b + 2a = 5a + (3b + 2a)$ _______
$= 5a + (2a + 3b)$ _______
$= (5a + 2a) + 3b$_______
$= 7a + 3b$ ____________

d) $4[3+2(x+5)] = 12 + 8(x+5)$ ____________

$= 12 + (8x + 40)$ ____________

$= 12 + (40 + 8x)$ ____________

$= (12 + 40) + 8x$ ____________

$= 52 + 8x$ ____________

e) $x(x+3) + 2(x+5)$

$= x^2 + x \times 3 + (2x + 10)$ ____________

$= (x^2 + 3x) + (2x + 10)$ ____________

$= [x^2 + (3x + 2x)] + 10$ ____________

$= [x^2 + (3+2)x] + 10$ ____________

$= x^2 + 5x + 10$ ____________

f) $-5 + 2(3x + 4)$

$= -5 + [2(3x) + 2(4)]$ ____________

$= -5 + [6x + 8]$ ____________

$= 6x + [-5+8]$ ____________

$= 6x + 3$ ____________

1.3.4 Division of Rational Numbers

Activity 1.9

Divide and write each answer in lowest terms.

a) $\frac{3}{7} \div \frac{5}{8}$

b) $\frac{11}{5} \div \frac{1}{5}$

c) $\frac{3}{8} \div \frac{5}{3}$

d) $9 \div \frac{4}{5}$

e) $\frac{3}{8} \div \frac{1}{6}$

f) $\frac{4}{9} \div \frac{2}{8}$

Multiplication and **division** are inverse operations of each other in the set of non- zero rational numbers. To divide 12 by 3 is to find a number, which gives the product 12 when multiplied by 3. This number is 4. Thus $12 \div 3 = 4$ because $4 \times 3 = 12$.

- ✓ The symbol "÷" denotes the operation of **division** and it is read as **divided by** so, $12 \div 3$ is read as 12 divided by 3.
- ✓ In the division $12 \div 3 = 4$, 12 is called the **dividend**, 3 is called the **divisor** and 4 which is the result of the division is called the **quotient.**

You may define division as follows.

Division

For any numbers a, b and c where $b \neq 0$, $a \div b = c$, if and only if $c \times b = a$.

- ✓ c or $a \div b$ is the **quotient** obtained by dividing a by b.
- ✓ $a \div b$ is read as a is divided by b.
- ✓ In the division $a \div b = c$ the number 'a' is called the **dividend.**
- ✓ 'b' is called the **divisor** and 'c' is called **quotient.**
- ✓ The quotient $a \div b$ is also denoted by $\frac{a}{b}$ or a/b.

Based on the above information, you can easily find out rules for the division of rational numbers analogous to those of multiplication.

***Rule*:** The rules for division of two rational numbers:

1. To determine the sign of the quotient:
   a) If the sign of the dividend and the divisor are the same, the sign of the quotient is (+).
   b) If the sign of the dividend and the divisor are different, the sign of the quotient is (-).
2. Determination of the values of the quotient:
   Divide the absolute value of the dividend by the divisor.

Example 38.

Look at Table 1.6 below:

	Problem	Divisor and dividend	Absolute value	Quotient
a	**28÷4**	**Both positive (+)**	**28 ÷4 = 7**	**7**
b	**-2.8 ÷ -0.2**	**Both negative (-)**	**2.8 ÷0.2 = 14**	**14**
c	**-10÷2**	**One negative and one positive**	**10÷2 = 5**	**-5**
d	**4.8 ÷ (-4)**	**One positive and one negative**	**4.8 ÷4 = 1.2**	**-1.2**
e	**0÷10**	**Dividend 0**	**0÷10= 0**	**0**
f	**0÷(-10)**	**Dividend 0**	**0÷10 = 0**	**0**

Note: Division by zero is not defined under the set of rational numbers.

CAUTION

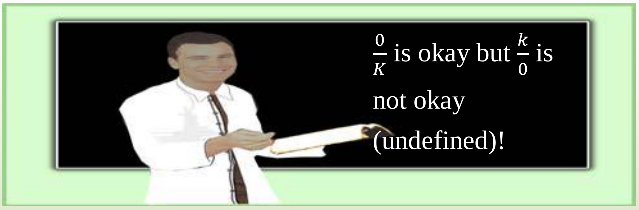


Figure 1.29

For any rational numbers a and b, $\frac{a}{b} = a \div b$ (a, b $\in \mathbb{Q}$, b $\neq 0$)

Remember that, $\frac{3}{4}$ is the reciprocal of $\frac{4}{3}$.

Remember that, $\frac{1}{5}$ is the reciprocal of 5.

Remember that, $\frac{-7}{11}$ is the reciprocal of $\frac{-11}{7}$.

Dividing a given rational number (the dividend) by another non- zero rational number (the divisor) means multiplying the dividend by the reciprocal of the divisor.

Note: For any two rational numbers, $\frac{a}{b}$ and $\frac{c}{d}$ where b, c and d $\neq 0$;

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc} \text{ where } \frac{d}{c} \text{ is the reciprocal of } \frac{c}{d}.$$

Example 39. Compute: a) $\frac{9}{10} \div \frac{5}{7}$ b) $-6 \div \frac{11}{13}$ c) $\frac{-7}{9} \div -2$

Solution:

a) $\frac{9}{10} \div \frac{5}{7} = \frac{9}{10} \times \frac{7}{5} = \frac{63}{50}$

b) $-6 \div \frac{11}{13} = -6 \times \frac{13}{11} = \frac{-78}{11}$

c) $\frac{-7}{9} \div (-2) = \frac{-7}{9} \times \frac{-1}{2} = \frac{7}{18}$

Exercise 1k

1. Divide

a) $\frac{3}{5} \div \left(\frac{-6}{15}\right)$ c) $\frac{-4}{11} \div \left(\frac{-4}{11}\right)$ e) $-8 \div \left(\frac{-16}{21}\right)$ g) $0 \div \frac{3}{5}$

b) $\frac{-4}{7} \div \frac{3}{14}$ d) $\frac{-5}{16} \div \frac{11}{8}$ f) $\frac{-14}{15} \div (-7)$ h) $\frac{-27}{3} \div 0$

2. Compute

a) 4. 6 ÷ (-6) d) 90 × (-8) + 100 ÷(-50)

b) 12 × 4 ÷6× (-8) e) – 0.2× (-0.3) + (0. 8×(-0.7)

c) 9 × (-8) ÷72(-2)

3. Reduce to the lowest term if possible:

a) $\frac{-54}{72}$ c) $\frac{-48}{-120}$ e) 245 ÷ 10

f) 79.2 ÷ 10

b) $\frac{50}{-80}$ d) $\frac{-2a^2b^2}{b}(b \neq 0)$

4. Solve the following equations.

a) 2y × (- 28) = 48 c) $\frac{1}{2}y = -8$ e) $-2x = \frac{0}{10}$

b) 3y ÷ (-2) = 24 d) 5x + 10 = - 30 f) $\frac{2}{3}x = \frac{-2}{27}$

5. Simplify

a) $\left(\frac{-18}{5} \div \frac{9}{35}\right) \times \left(\frac{-3}{7}\right)$ c) $\left[1\frac{2}{3} \times 4\frac{2}{3}\right] \div 6\frac{1}{9}$

b) $\left[\frac{-12}{25} \times \left(\frac{-5}{7}\right)\right] \div \left(\frac{-9}{14}\right)$ d) $\left(5\frac{1}{16} \div 6\frac{3}{4}\right) \times \left(7\frac{5}{9}\right)$

Challenge Problems

6. Find the quotient. Think of a simpler problem and use the pattern to solve the problem: $\frac{1}{2} \div \frac{1}{2} \div \frac{1}{2} \div \frac{1}{2} \div \frac{1}{2} \div \frac{1}{2} \div \frac{1}{2}$.
7. Does (56 ÷ 8) ÷ 2 equal 56 ÷ (8 ÷ 2)? Is division associative.
8. Find the quotient of $(8x^2 + 20xy) \div 4x$.

Summary For Unit 1

1. The sum and product of two whole numbers are always a whole number.
2. The sum, difference and product of two or more integers are always an integer.
3. The set of rational numbers is defined as:

   $\mathbb{Q} = \left\{\frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{Z} \text{ and } b \neq 0\right\}$.
4. $\mathbb{Z} = \mathbb{Z}^{+} \cup \{ 0 \} \cup \mathbb{Z}^{-}$
5. $\mathbb{Q} = \mathbb{Q}^{+} \cup \{ 0 \} \cup \mathbb{Q}^{-}$
6. The absolute value of a rational number X is denoted by the symbol |x| and defined as:

   $$|X| = \begin{cases} x \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -x \text{ if } x < 0 \end{cases}$$
7. Subtraction of any rational number can be treated as the inverse operation of addition.
8. The sum of two opposite rational numbers is 0.
9. Rules of signs for Addition:

   Let a and b be rational numbers:
   a) Negative plus negative equals negative:- $a+(-b) = -(a+b)$.
   b) Positive plus negative equals positive if $a > b$: $a + (-b) = a - b$ is positive.
   c) Positive plus negative equals negative if $a < b$: $a + (-b) = -(b - a)$ is negative.
10. Rules of signs for Multiplication

    Let a and b be rational numbers:
    a) Positive times negative equals negative: $a\times(-b) = -(a\times b)$.
    b) Negative times positive equals negative :$-a \times b = -(a\times b)$.
    c) Negative times negative equals positive $-a\times(-b) = a\times b$.
11. Rules of signs for Division

    Let a and b be rational numbers:
    a) Positive divided by negative equals negative: $a \div -b = -(a\div b)$.
    b) Negative divided by positive equals negative: $-a \div b = -(a \div b)$.
    c) Negative divided by negative equals positive: $-a \div (-b) = a \div b$.

Miscellaneous Exercise 1

1. Decide whether each of the following is true or false.

   a) $0 > -100$ c) $-10,000 > 10,000$ e) $|-2.9| > 2.6$

   b) $3\frac{1}{2} < \frac{0}{10}$ d) $|2.6| < 2.6$ f) $|98.6| = |-98.6|$

2. Evaluate:

   a) $-4(5-(36 \div 4))$ c) $3\frac{1}{5} + \left(\frac{-7}{8}\right)$

   b) $10-(5-(4-(8-2))$ d) $\frac{-1}{4} + \left(\frac{-5}{9}\right)$

3. Simplify by combining Like terms.

   a) $3k - 2k$ d) $2x^2 + 5x - 4x^2 + x - x^2$

   b) $5x^2 - 10x - 8x^2 + x$ e) $(3x+y) + x$

   c) $-(m+n) + 2(m-3n)$ f) $2(5+x) + 4(5+x)$

4. Simplify each of the following expression.

   a) $3x + 2(7x+5)$ d) $-7(-2(3x+1)+4)+9$

   b) $-5 + 2(3x+4)$ e) $3x^2 + 2(5x + 3x^2)$

   c) $-2(-3) + (4(-3) + 5(2))$ f) $\frac{3}{8}(y+2) - \frac{1}{4}(y-2)$

5. Find the simplified form of $\left[\left(\frac{1}{2}+\frac{1}{3}\right)\times\frac{1}{4}\right] \div \left[\left(\frac{2}{5}+\frac{3}{4}\right)\div\frac{6}{12}\right]$.

6. Find the simplified form of $\left[\frac{-24}{5}\times\frac{15}{16}\right] \div \left[\frac{6}{4}\times\frac{-12}{8}\right]$.

7. Simplify the following expression.

   a) $\dfrac{\frac{1}{8} \div \left(\frac{1}{4}-\frac{1}{3}\right)}{\frac{4}{3}+\frac{1}{2}\left[\frac{1}{2}\div\frac{3}{2}\right]}$ c) $\left[2\frac{3}{4} + 4\frac{1}{8} \times 1\frac{5}{11}\right] \div \left[7\frac{7}{8} \div 10\frac{7}{20}\right]$

   b) $\dfrac{\frac{15}{37}\left[\frac{4}{15}+\frac{23}{30}-\frac{5}{12}\right]}{\frac{1}{7}\left[\frac{14}{3}\div\frac{1}{3}\right]}$ d) $\dfrac{\frac{5}{11}\left[\frac{2}{3}-\frac{1}{5}\right]}{\frac{7}{2}+\left[1+\frac{5}{6}\right]}$

8. Solve each of the following absolute value equations.

a) $|2y - 4| = 12$

b) $|3x + 2| = 7$

c) $3|4x - 1| - 5 = 10$

d) $|2x + 15| = -10$

9. If $x = -6$ and $y = 10$, then find $\frac{|x| - |3y|}{|xy|}$.

10. Evaluate the following expression.

a) $\frac{x}{2} + 11$ **when** $x = 10$

b) 7x – 4y when x = 10 and y = $\frac{1}{2}$

c) $3x^2 + 6y^2$ **when x = 0 and y = 2**

11. Solve for n: $\frac{n}{18} = \frac{7}{9}$.

12. In the expression 8÷ 2 = 4 the dividend is __?__ the divisor is __?__and the quaint is __?__ .

13. Multiply

a) $\left[4\frac{3}{4} + \left(1\frac{1}{2}\right)\right] \times \left[6\frac{1}{8} + \left(5\frac{3}{8}\right)\right]$

b) $(2.01 + (-3.17)) \times (-4.2 + 17.8)$

c) $4\frac{3}{4} \times \left[\frac{-16}{15} \times (-3.25)\right]$

d) $\frac{5}{16} \times \left[\frac{4}{15} \times \left(\frac{-4}{3}\right)\right]$

14. Adding 3 to some number, then multiplying the result by 7 gives 28. What was the original number?

15. Some number is added to itself. The result is multiplied by 5 and the product is 15. What was the number?

UNIT 2

LINEAR EQUATIONS AND INEQUALITIES

Unit outcomes:

After completing this unit, you should be able to:

- solve linear equations using transformation rules.
- solve linear inequalities using transformation rules.

Introduction

Based on your knowledge of working with variables and solving one step of linear equations and inequalities. You will learn more about solving linear equations and inequalities involving more than one steps. When you do this you will apply the rules of equivalent transformations of equations and inequalities appropriately.

2.1. Solving Linear Equations

Group Work 2.1

Discuss with your friends

1. Explain each of the following key terms, and give your own example.
    a. Term, like terms or similar terms.
    b. Coefficient of a term.
    c. Algebraic expressions.
    d. Equation.
    e. Equivalent equation.
2. Give examples of your own for:
    a. like terms or similar term
    b. unlike terms
    c. equation
    d. algebraic expressions
3. What are the numerical coefficients of x^3 and $-y^3$?

Definition 2.1: A constant (a number), a variable or product of a number and variable is called a term.

Example 1: 2, $\frac{-3}{2}$, x, 3x, $-4x^2$ are called terms.

Consider Group A and Group B

Group A
5x and -20x
$-80a^2b^2$ and $\frac{-1}{2}a^2b^2$
$6x^2$ and $70x^2$

Group B
$-10a^2b^2$ and $12c^2d^2$
20xy and abcd
5ab and 6xy

? In general how do you see the differences between Group A and Group B? Discuss the differences with your teacher orally.

Definition 2.2: Like terms or similar terms are terms whose variables and exponents of variables are exactly the same but differ only in their numerical coefficients.

Note: Terms that are not like terms are called unlike terms.

Example 2: Terms like $-10a^2$, $170a^2$ and a^2 are like terms. Because they have the same variables with equal exponents but differ only in their numerical coefficients.

Example 3: Terms like -5ab and $7x^2y^2$ are unlike terms. Because they do not have the same variables.

Definition 2.3: In the product of a number and variable, the factor which is a numerical constant of a term is called a numerical coefficient.

Example 4: In each of the following expression, determine the numerical coefficient.

a. 56b b. $\frac{-5}{2}a^2b^2$ c. $\frac{-1}{4}xy$ d. $-x^2$

Solution:

a. The numerical coefficient of 56b is 56.

b. The numerical coefficient of $\frac{-5}{2}a^2b^2$ is $\frac{-5}{2}$.

c. The numerical coefficient of $\frac{-1}{4}xy$ is $\frac{-1}{4}$.

d. The numerical coefficient of $-x^2$ is -1.

Consider Group C and Group D

Group C
2x - 3
5y
a + 2b + 3c
2(ℓ+w)

Group D
2x – 3 = 10
5y = 60
a + 2b + 3c = 100
P = 2(ℓ+w)

? Do you observe the differences between Group A and group B? Discuss the differences with your teacher.

Definition 2.4: An equation is a mathematical statement in which two algebraic expressions are joined by equality sign. Therefore, an equation must contain an equal sign,=.

Example 5. Some examples of equations are:

a. $\frac{5}{2}x - 10 = 40$

b. $4x + 10 = 3\frac{1}{2}$

c. $3\frac{1}{2}x - 5\frac{3}{2} = 10$

d. $\frac{1}{2}x + \frac{2}{5}x - 10x = 50$

Note: Algebraic expressions have only one side.

- Algebraic expressions are formed by using numbers, letters (variables) and the basic operations of addition, subtraction, multiplication, and division.

Example 6. Some examples of algebraic expressions are:

a. $2x-4$

b. $\frac{\pi}{2} + \frac{1}{2}|-5x|$

c. $\left(\frac{-6}{5}\right) + \frac{x}{3} + 20$

d. $\left(-5\frac{1}{2}\right) \div \frac{\pi}{2}$

e. 215

f. 3x

Exercise 2A

1. State whether each of the following is an equation or an algebraic expression.

   a. $2x+10=5x+60$

   b. $|2x+10|$

   c. $10+3.8= 14.78x-10$

   d. $9x + 10=5x$

2. In each of the following expressions, determine the numerical coefficient.

   a. $\frac{3}{2}x^4$

   b. $-3\frac{1}{2}x^2$

   c. $\frac{-2}{3}x^2y^2$

   d. $\frac{-2}{7}x^5$

3. Identify whether each pair of the following algebraic expressions are like terms or unlike terms.

a. $\frac{3}{5}a^5b^2$ and $\frac{-5}{2}b^2a^5$

b. $3\frac{5}{6}xy$ and $3\frac{5}{6}x^2y^2$

c. -80abc and abc

d. $a^2b^2c^2d^2$ and $a^4b^4c^4d^4$

Challenge Problems

4. $0.0056x+26=100x+3\frac{1}{2}$ is a linear equation. Explain the main reason with your partner.
5. $a^5b^5c^5d^5$ and $-2(a^5b^5c^5d^5)$ are like terms. State the reason with your teacher orally.

2.1.1 Rules of Transformation for Equation

The following are basic rules of equality (=) that are used to get equivalent equations in solving a given equation.

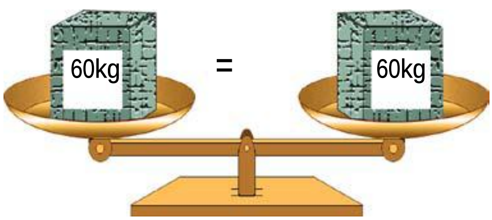
Rule 1: For all rational numbers a, b and c

a. If an equation a =b is true, then a + c = b + c is true for any rational number c.

b. If an equation a =b is true, then a-c =b-c is true for any rational number c.

Addition and subtraction properties of equality indicate that adding or subtracting the same quantity to each side of an equation results in an equivalent equation. This is true because if two quantities are increased or decreased by the same amount, then the resulting quantities will also be equal see Figure 2.1 below.

a)



b)

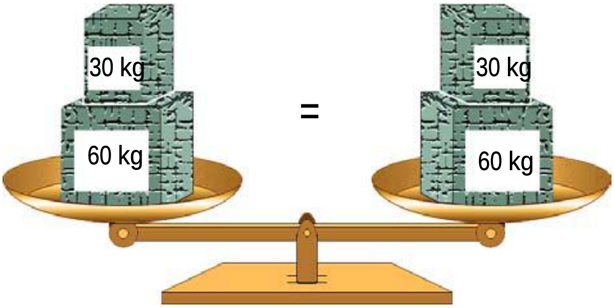


Figure 2.1 Balance

Rule 2: For all rational numbers a, b and c where $c \neq 0$, and

a. If an equation a=b is true, then ac = bc is true for any rational number c.

b. If an equation a=b is true, then $\frac{a}{c} = \frac{b}{c}$ is true for any rational number c.

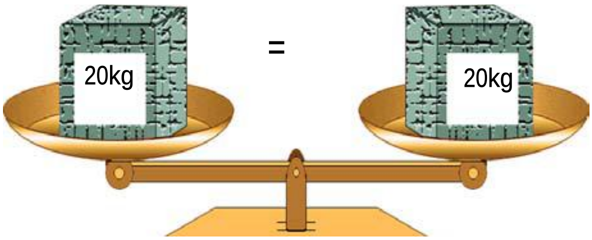
To understand the multiplication property of equality, consider the following example. Suppose you start with a true equation such as 20=20. If both sides of an equations are multiplied by a constant such as 2 the result is also a true statement, see Figure 2.2 below.

a)

20 = 20

2×20 = 2×20

40 = 40



b)

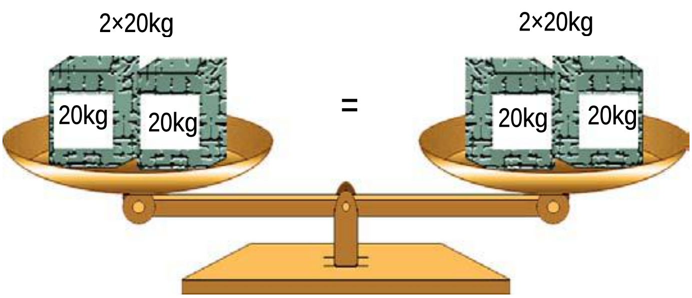


Figure 2.2 balance

Similarly, if an equation is divided by a non zero real numbers such as 2, the result is also a true statement, see Figure 2.3 below.

$$20 = 20$$

$$\frac{20}{2} = \frac{20}{2}$$

$$10 = 10$$

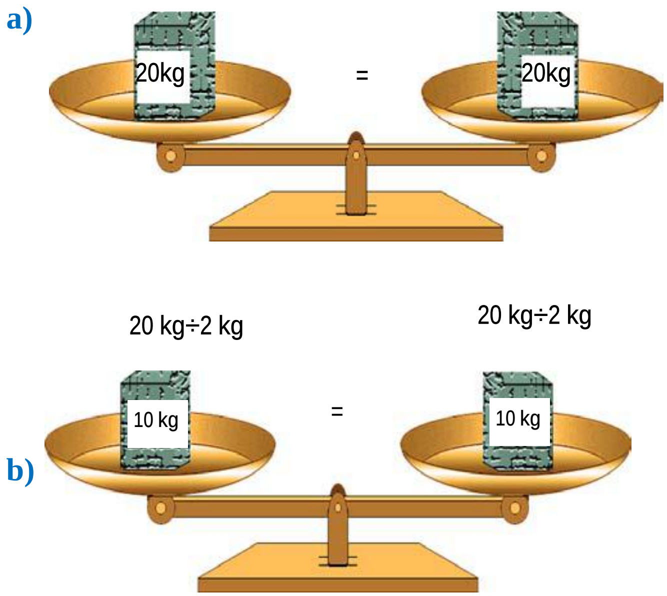


Figure 2.3 Balance

Example 7: To find X from X + 60 = 90

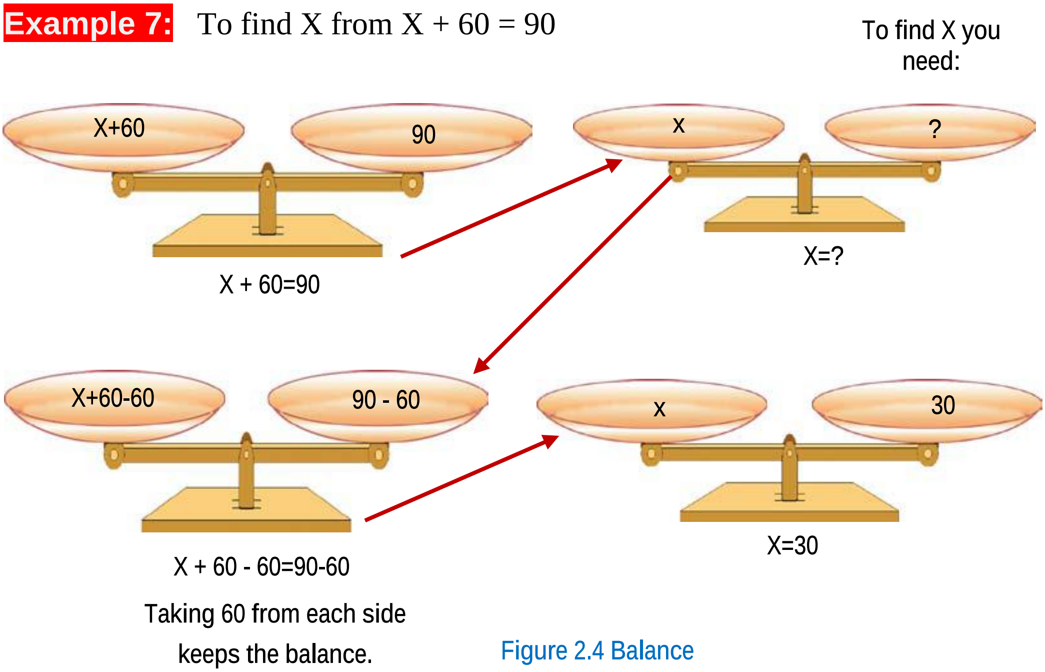


Figure 2.4 Balance

Example 8: Solve each of the following equations by using addition rules.

a. $x+\frac{3}{5}=\frac{8}{5}$ b. x -6 = -20

Solution:

a. $x+\frac{3}{5}=\frac{8}{5}$ . . . . . Given equation

$x+\frac{3}{5}+\left(\frac{-3}{5}\right)=\frac{8}{5}+\left(\frac{-3}{5}\right)$.......... Adding $\frac{-3}{5}$ on both sides.

x+0 = 1……..Simplifying

x = 1 ……. x is solved

✓ **Check:** When x = 1

$x+\frac{3}{5}=\frac{8}{5}$

$1+\frac{3}{5}\stackrel{?}{=}\frac{8}{5}$

$\frac{8}{5}=\frac{8}{5}$ .... True

Since $\frac{8}{5}=\frac{8}{5}$ is a true statement, x=1.

b. x - 6 =-20…….Given equation

x - 6 + 6=-20+6…….Adding 6 on both sides.

x + 0= -14……Simplifying

x = -14 …. x is solved

✓ **Check**: when x = -14

x-6 = -20

$-14-6 \stackrel{?}{=} -20$

- 20 = -20 ….True

Since -20 = -20 is a true statement, x= -14

Example 9: Solve each of the following equations by using multiplication rules.

a. 8x = 72 b. $\frac{-4}{5}x=10$

Solution:

a. $8x = 72$ …..Given equation

$\frac{1}{8} \times 8x = \frac{1}{8} \times 72$ .... Multiplying by $\frac{1}{8}$ on both sides

$1 \times x = 9$………Simplifying

$x = 9$ ……..x is solved

✓ **Check**: When $x = 9$

$8x = 72$

$8 \times 9 \stackrel{?}{=} 72$

$72 = 72$…….True

Since $72 = 72$ is a true statement, $x = 9$

b. $\frac{-4}{5}x = 40$ …….Given equation

$\left(\frac{-5}{4}\right) \times \left(\frac{-4}{5}x\right) = \left(\frac{-5}{4}\right) \times 40$ …..Multiplying by $\frac{-5}{4}$ on both sides.

$1 \times x = -50$……..Simplifying

$x = -50$ ……..x is solved

✓ **Check:** When $x = -50$

$\frac{-4}{5}x = 40$

$\left(\frac{-4}{5}\right) \times -50 \stackrel{?}{=} 40$

$40 = 40$ ……True

Since $40 = 40$ is a true statement, $x = -50$.

2.1.2 Linear Equations in One Variable

? Consider the equation $3x+5 = 0$, $\frac{-3}{5}\mathbf{x} - 10 = \frac{3}{4}$, $\frac{1}{2}\mathbf{x} + 10 = 0$ etc are examples of linear equations. Why? Discuss the reason with your teacher in the class.

Definition 2.5: A linear equation in one variable x is an equation which can be written in standard form $ax + b = 0$, where a and b are constant numbers with $a \neq 0$.

From this definition, you can deduce that an equation of a single variable in which the highest exponent of the variable involved is one is called **a linear equation**.

Example 10: Which of the following equations are linear and which are not linear.

a. $5x+3\frac{5}{6}=10$

c. $3x^2-\frac{8x^2}{2}+10=0$

b. $\frac{-3}{2}x+20=10-\frac{1}{2}x$

d. $2x^2+2x=10$

Solution: a and b are linear equations. Because the highest exponent of the variable is one, but c and d are not linear equations. Why?

Briefly, all equations have two sides; with respect to the equality sign called left hand sides (L.H.S) and right hand sides (R.H.S) of the equality sign. These two sides are equal to each other like that of a simple balance. Thus equation is just a simple balance as shown in Figure 2.5 below.

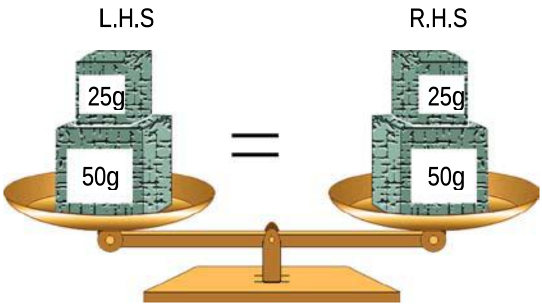


Figure 2.5 Simple balance

Note that the symbol " = " is read as "equals" or is "equal to".

Example 11: Identify the L.H.S and R.H.S of the following linear equations.

a. $\frac{5}{6}x+10=5$

b. 3y-16=19-6y

Solution:

L.H.S	R.H.S
$\frac{5}{6}x+10$	5
3y-16	19-6y

Solving an equation means, applying the appropriate transformation rules to get a simplified equivalent equation in which the variable alone appears at one side and a constant (number) on the other side of the equality sign "=".

This constant number is called the **solution of the given equation.**

Note: Linear equations have exactly one solution. To see this, consider the following steps.

$ax + b = 0$ ……Given equation

$ax + b+(-b) = 0+(-b)$……Adding –b on both sides.

$ax = -b$ ….Simplifying

$\frac{ax}{a} = \frac{-b}{a}$…… Dividing both sides by a (since $a \neq 0$).

$x = \frac{-b}{a}$…… Simplifying

Thus, the equation $ax+b = 0$ has exactly one solution, that is $x = \frac{-b}{a}$.

Activity 2.1

1. Solve each of the following equations and mention the rules of transformation together.

   a. $0.8 + 2x = 3.5 – 0.5x$

   b. $8x – (3x – 5)=40$

   c. $(2x + 8) – 20 = -(3x – 18)$

   d. $5x – 17 – 2x = 6x – 1 – x$

2. $ax^2+bx+c=0$ is not a linear equation. Discuss the reason with your teacher in the class.

Example 12: Solve each of the following equations, in doing so indicate the rules you used.

a. $12x-14= 4x+10$

b. $3(7-2x)= 14 -8 (x-1)$

c. $8x+6-2x= -12-4x+5$

d. $7x-3(2x-5)= 6(2+3x)-31$

Solution:

a. $12x-14 = 4x+10$.....Given equation

$12x-14+14 = 4x+10+14$.......Adding 14 to both sides.

$12x = 4x+24$.........Simplifying.

$12x-4x = 4x-4x+24$.........Subtracting 4x from both sides.

$8x = 24$ ....Simplifying.

$\frac{1}{8}\times 8x = \frac{1}{8}\times 24$...........Multiplying both sides by $\frac{1}{8}$.

$1\times x = 3$ ......Simplifying

$x = 3$ ....x is solved.

✓ Check: When $x = 3$

$12x - 14 = 4x + 10$

$12\times 3 - 14 \stackrel{?}{=} 4\times 3 + 10$

$36 - 14 \stackrel{?}{=} 12 + 10$

$22 = 22$ .....True

Since $22 = 22$ is a true statement, $x = 3$

b. $3(7-2x) = 14-8(x-1)$.......Given equation

$21 - 6x = 14 - 8x + 8$........Removing parentheses by distributive property.

$21 - 6x = 22 - 8x$.......Simplifying.

$21 - 6x + 6x = 22 - 8x + 6x$.......Adding 6x to both sides.

$21 = 22 - 2x$.........Simplifying

$21 - 22 = 22 - 22 - 2x$.....Subtracting 22 from both sides

$-1 = -2x$ ......Simplifying

$-1\times\left(\frac{-1}{2}\right) = -2x\left(\frac{-1}{2}\right)$........... Multiplying both sides by $\left(\frac{-1}{2}\right)$.

$x = \frac{1}{2}$

✓ Check: When $x = \frac{1}{2}$

$3(7 - 2x) = 14 - 8(x - 1)$

$3\left(7 - 2\times\frac{1}{2}\right) \stackrel{?}{=} 14 - 8\left(\frac{1}{2} - 1\right)$

$3(7 - 1) = 14 - 8\left(\frac{-1}{2}\right)$

$3(6) = 14 + 4$

$18 = 18$ ......True

Since $18 = 18$ is a true statement, $x = \frac{1}{2}$ is the solution of the given equation.

c. $8x + 6 - 2x = -12 - 4x + 5$…….Given equation

$8x - 2x + 6 + (-6) = -12 + 5 + (-6) - 4x$……. Adding -6 on both sides.

$6x = -13 - 4x$…..Simplifying

$6x + 4x = -13 - 4x + 4x$……Adding 4x to both sides.

$10x = -13$……Simplifying

$\frac{1}{10}\times 10x = \frac{1}{10}\times(-13)$........Multiplying by $\frac{1}{10}$ both sides.

$1 \times x = \frac{-13}{10}$ …...Simplifying

$x = \frac{-13}{10}$ .... x is solved

✓ Check: When $x = \frac{-13}{10}$

$8x + 6 - 2x = -12 - 4x + 5$

$$8\left(\frac{-13}{10}\right) + 6 - 2\left(\frac{-13}{10}\right) \overset{?}{=} -12 - 4\left(\frac{-13}{10}\right) + 5$$

$$\frac{-52}{5} + 6 + \frac{13}{5} \overset{?}{=} -12 + \frac{26}{5} + 5$$

$$\frac{-39}{5} + 6 \overset{?}{=} -7 + \frac{26}{5}$$

$$\frac{-9}{5} = \frac{-9}{5} \text{........... True}$$

Since $\frac{-9}{5} = \frac{-9}{5}$ is true statement, $x = \frac{-13}{10}$. or simply $\frac{-13}{10}$ is the solution of the given equation.

d. $7x - 3(2x - 5) = 6(2 + 3x) - 31$…..Given equation

$7x - 6x + 15 = 12 + 18x - 31$…..Distributive property.

$x + 15 = 18x - 19$ ……..Simplifying

$x + (-x) + 15 = 18x + (-x) - 19$…….Subtracting x from both sides.

$15 = 17x - 19$…….Simplifying

$15 + 19 = 17x - 19 + 19$…….Adding 19 from both sides.

$34 = 17x$……..Simplifying

$\frac{34}{17} = \frac{17x}{17}$ …….Dividing both sides by 17

$x = 2$…….. x is solved

✓ Check: When x =2

$7x - 3(2x - 5) = 6(2 + 3x) - 31$

$7\times2-3(2\times 2 - 5) \underset{=}{?} 6 (2 +3 \times 2) -31$

$14 - 3(-1) \underset{=}{?} 6 (8)- 31$

$14 + 3 \underset{=}{?} 48 -31$

17= 17 …..True

Since 17 = 17 is true statement, x = 2

The set that contains the solution of a given equation is called the **solution set of the equation.**

> **Definition 2.6:** Two equations are said to be equivalent if and only if they have exactly the same solution set.

Example 13: show that $2[9 - (x - 3) + 4x] = 4x - 5(x + 2) - 8$ and $\frac{x}{2} = -3$ are equivalent equations.

Solution: $2 [9 - (x-3) + 4x] = 4x - 5 (x+2) - 8$ .... Given equation

$2 [9 - x + 3 +4x] = 4x - 5x -10 - 8$...................... Remove parentheses.

$2 [12 + 3x]=-x-18$……..Combine like terms

$24+6x= -x-18$………Remove parentheses

$24+6x+x= -x+x-18$….. Add x to both sides.

$24+7x= -18$…….Simplifying

$24-24+7x= -18-24$……Subtract 24 from both sides.

$7x= -42$…….Simplifying

$\frac{7x}{7} = \frac{-42}{7}$ .... Dividing both sides by 7.

$x = -6$ ……. X is solved

And $\frac{x}{2} = -3$ .... Given equation

$x= -6$ ….. Multiplying both sides by 2

Therefore: $2 [9 - (x-3] +4x] = 4x-5 (x+2) -8$ and

$\frac{x}{2} = -3$ are equivalent equations.

Exercise 2B

1. Which of the following pairs of equations are equivalent?
   a. $2x+8=18$ and $2x=18-12$
   b. $9x-\frac{9}{8}=\frac{9}{4}$ and $9x=\frac{9}{8}$
   c. $21x=38$ and $3x=36$
   d. $2x+(-6)=14$ and $2x=14+6$
   e. $3x=182$ and $x=\frac{182}{6}$
   f. $\frac{3}{5}x-\frac{3}{7}=10$ and $21x-365=0$
2. Show that $4(2x-1)=3(x+1)-2$ and $8x=3x+5$ are equivalent equations.
3. Solve the following linear equations and finally check your answers.
   a. $3x-9=4x+5$
   b. $2(3x+4)=6-(2x-5)$
   c. $2\left(\frac{x-3}{5}\right)=x-\frac{3}{5}$
   d. $2(2x+1)=3(x+3)+x-6$
   e. $270 \div x=540;\ x\neq 0$
   f. $4(2x-1)+6=7x-3(x+2)$
4. Show that $\frac{2}{3}(x+4)+\frac{3}{5}(2x+1)=0$ and $4(x+4)-3(2-x)=17$ are not equivalent equation.

Challenge Problems

5. Solve for x
   a. $ax+b=cx+d;\ a\neq c$
   b. $m(x-n)=3(r-x);\ m\neq 3$
   c. $ax+b=c;\ a\neq 0$
   d. $x+y=b(y-x);\ b\neq -1$
   e. $a_1x+b_1y=a_2x+b_2y;\ a_1\neq a_2$

2.1.3 Some Word problems

Group Work 2.2

1. Translate the algebraic expression x+12 in five different word phrases.
2. Translate the algebraic expression x–7 in six different word phrases.
3. Translate the algebraic expression 4x in four different word phrases.
4. Translate the algebraic expression $\frac{x}{6}$ in four different word phrases.

In this topic, you will apply the knowledge acquired on equations. The connection between an unknown number and other numbers which are known (constant) often arise out of practical life. To solve such problems verbal sentences needed to be changed into mathematical sentences. Relationship between such numbers or quantities given in word problem need to be expressed in the form of equation. You

will now demonstrate how to solve a word problem by changing into mathematical equation.

Example14: Translate the algebraic expression x+10 in different word phrases.

Solution:

Word phrases	Algebraic expression (or symbols)
• A number plus ten. • The sum of a number and ten. • Ten added to a number. • A number increased by ten. • Ten more than a number.	x + 10

Example15: Translate the algebraic expression $\frac{x}{7}$ in different word phrases.

Solution:

Word phrases	Algebraic expression (or symbols)
• A number divide by seven. • The quotient of a number and seven. • The ratio of a number to seven. • one-seventh of a number.	$\frac{x}{7}$

Example 16: **(Relationship between temperature scales)**

Celsius scale

Fahrenheit scale

The Celsius and Fahrenheit temperature scales are shown on thermometer in Figure2.6. The relationship between the temperature readings C and F is given by $C=\frac{5}{9}(F-32)$.(Express F in terms of C).

Solution: To solve for F you must obtain a formula that has F by itself on one side of the equals sign. You may do this as follows:

Figure 2.6 Temperature scales

$$C = \frac{5}{9}(F-32) \quad \text{.........Given equation}$$

$$\frac{9}{5}C = F-32 \text{ ....Multiply both sides by } \frac{9}{5}$$

$$\frac{9}{5}C+32 = F \text{.......... Adding 32 from both sides.}$$

$$F = \frac{9}{5}C+32$$

Example 17: **(Test average)**

A student take a mathematics test scores of 64 and 78. What score on a third test will give the student an average of 80?

Solution:

The unknown quantity is the score on the third test,
so you let x = score on the third test.
The average scores will be calculated on 64, 78 and x.

Thus average score = $\dfrac{64+78+x}{3}$

$$\frac{64+78+x}{3} = 80$$

$$64 + 78+x = 80 \times 3 \text{ ....Multiplying both sides by 3}$$

$$142+x= 240 \text{.........Simplify}$$

$$x = 98 \text{......... x is solved}$$

✓ **Check:** If the three test scores are 64, 78 and 98, then the average is $\dfrac{64+78+98}{3} = \dfrac{240}{3} = 80.$

Example 18: **(Age problem)**

The sum of the ages of a man and his wife is 96 years. The man is 6 years older than his wife. How old is his wife?

Solution: let m = man age and w = wife age, then

m+w = 96 …….Translated equation(1)

m = 6+w ……..Translated equation (2)

6+w+w = 96……..Substituting equation (2) into(1)

6+2w = 96-6……Combine like terms

2w+6-6 = 96-6………Subtracting 6 both sides.

2w = 90 ………Simplifying

$\frac{2w}{2} = \frac{90}{2}$ ……………Divides both sides by 2

w = 45 …….w is solved

Therefore, the age of his wife is 45 years old.

Exercise 2C

Solve each of the following word problems.

1. If three fourth of a number is one-tenths, what is the number?
2. The sum of two consecutive integers is three times their difference. What is the larger number?
3. Can you find a number that satisfy the following property?
   a. If you multiply the number by 2 and add 4, the result you get will be the same as three times the number decreased by 7.
   b. If you increase the number by 4 and double this sum, the result you get will be the same as four times the number decreases by 6.
4. In a class there are 48 students. The number of girls is 3 times the number of boys. How many boys and how many girls are there in the class?
5. A farmer has sheep and hen. The sheep and hens together have 100 heads and 356 legs. How many sheep and hens does the farmer have?
6. 8 times a certain number is added to 5 times a second number to give 184. The first number minus the second number is -3. Find these numbers.
7. The perimeter of a rectangular field is 628m. The length of the field exceeds its width by 6m. Find the dimensions.

2.2. Solving Linear Inequalities

Activity 2.2

Discuss with your friends.

Solve each of the following linear Inequalities.

a. $2(5-x) \leq 3(1-2x) + 4$

b. $10(2x-4) \geq 12x-(2x+2)$

c. $8(2x-4)+6 \leq 14(2x+2)-12$

d. $0.5x + 0.5 > 0.2x + 2$

e. $0.7(x+3) < 0.4(x+3)$

From grade six mathematics lesson you have learnt about linear inequalities. Now in this sub topic you learn more about linear inequalities. The rules for transforming linear inequalities will be discussed in detail so as to find their solutions.

Definition 2.7: Mathematical sentence which contains one of the relation signs(symbols) $<, \leq, >, \geq$ or $\neq$ are called inequalities.

Example 19: Some examples of inequalities are:

a. $10x < 23$ b. $-2x > 5$ c. $\frac{1}{2}x \geq 4$ d. $\frac{3}{2}x \leq 10$

Definition 2.8: A linear inequality in one variable "**x**" is an inequality that can be written in the form of $ax + b < 0$, $ax + b \leq 0$ or $ax + b > 0$, $ax + b \geq 0$ where a and b are rational numbers and $a \neq 0$.

Example 20: Some examples of linear inequalities are:

a. $2x+10 > 0$

b. $5x+20 < 0$

c. $2x+12 \leq 0$

d. $6x+17 \geq 0$

2.2.1. Rules of Transformation for Inequalities

Group work 2.3

1. Solve each of the following inequalities by using the addition rule.

a. $x + 8 > 3$

b. $9x + 2.7 > 8x - 9.7$

c. $x - 0.35 \leq 0.25$

d. $x - 0.25 \geq -0.66$

2. **Solve each of the following inequalities by using the multiplication rule.**

   a . $1 - 3x \geq 6$　　　　c. $3x < 18$

   b. $81x \leq 3$　　　　d. $5 - x \leq 2x - 1$

The following rules are used to transform a given inequality to an equivalent inequality.

Rule 1: If the same number is added to or subtracted from both sides of an inequality, the direction of the inequality is unchanged. That is for any rational numbers a,b and c.

i. If $a < b$, then $a + c < b+c$.

ii. If $a < b$, then $a - c < b - c$.

Rule 2: If both sides of an inequality are multiplied or divide by the same positive number, the direction of the inequality is unchanged. That is for any rational numbers a,b and c.

i. If $a < b$ and $c > 0$, then $ac < bc$.

ii. If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$ (provided $c \neq 0$).

Rule 3: If both sides of an inequality are multiplied or divided by the same negative number, the direction of the inequality is reversed. That is for any rational numbers a, b and c.

i. If $a < b$ and $c < 0$, then $ac > bc$.

ii. If $a< b$ and $c < 0$, then, $\frac{a}{c} > \frac{b}{c}$ (provided $c \neq 0$).

Example 21: Solve each of the following inequalities by using the transformation rules.

a. $10x - 4 \leq 8x - 2$　　　　c. $4(x + 2) + 4 \leq 6(x + 1)-5$

b. $x+ \frac{3}{4} > \frac{3}{8}$　　　　d. $\frac{x+1}{3} \geq \frac{2x}{5} - 1$

Solution: a. $10x-4 \leq 8x-2$………Given inequality

$10x-4+4 \leq 8x-2+4$…….Adding 4 both sides

$10x \leq 8x+2$………Simplifying

$10x-8x \leq 8x-8x+2$…….Subtracting 8x from both sides

$2x \leq 2$……………….Simplifying

$\frac{2x}{2} \leq \frac{2}{2}$ ……………Dividing both sides by 2

$x \leq 1$…………….Simplifying

b. $x+\frac{3}{4} > \frac{3}{8}$ ……….Given inequality.

$x+\frac{3}{4}-\frac{3}{4} > \frac{3}{8}-\frac{3}{4}$ ………. Subtracting $\frac{3}{4}$ from both sides

$x > \frac{3}{8}-\frac{3}{4}$ ……… Simplifying

$x > \frac{3}{8}-\frac{3}{4}\times\frac{2}{2}$ ……………… Multiplying by $1 = \frac{2}{2}$

$x > \frac{3}{8}-\frac{6}{8}$ …………….. Simplifying

$x > \frac{-3}{8}$ ……………….. Solved

c. $4(x+2)+4 \leq 6(x+1)-5$……….Given inequality

$4x + 8+4 \leq 6x+6-5$……………Remove parenthesis by distributive property of "×" over "+"

$4x+12 \leq 6x+1$…………….Combine like terms.

$4x-6x+12 \leq 6x-6x+1$…….Subtracting 6x from both sides

$-2x+12 \leq 1$……………Simplifying

$-2x+12-12 \leq 1-12$………Subtracting 12 from both sides

$-2x \leq -11$……… Simplifying

$\frac{-2x}{-2} \geq -\frac{11}{-2}$ …….Dividing both sides by -2

$x \geq \frac{11}{2}$

d. $\frac{x+1}{3} \geq \frac{2x}{5}-1$…….Given inequality

$15\left(\frac{x+1}{3}\right) \geq 15(\frac{2x}{5}-1)$ …….Multiplying by 15 which is the LCM of the denominators 3 and 5

$5x+5 \geq 6x-15$……………..Remove parenthesis

$5x-6x+5 \geq 6x-6x-15$…………Subtracting 6x from both sides

$-x+5 \geq -15$…………Simplifying

$-x+5-5 \geq -15-5$…………Subtracting 5 from both sides

$-x \geq -20$…………..Simplifying

$x \leq 20$ …………. Solved

Definition 2.9: Two inequalities are said to be equivalent if and only if they have exactly the same solution set.

Example 22: Some examples of equivalent linear inequalities are:

a. $5x < 20$ and $x < 4$

b. $x > 3$ and $x+8 > 3+8$

c. $\frac{x}{2} < \frac{10}{6}$ and $6x < 20$

Exercise 2D

1. Which of the following pairs of inequalities are equivalent?

a. $2x - 6 > 4$ and $2x - 8 > 2$

b. $6x + 22 < 4$ and $6x < -14$

c. $3x + \frac{8}{12} < \frac{5}{12}$ and $36x + 8 < 5$

d. $3x + 8x + 21 \geq 0$ and $x \geq \frac{-21}{11}$

e. $\frac{4x}{3} < 12$ and $x < 9$

2. Identify whether each of the following inequalities is a linear inequality or not.

a. $6x + 6 > 3x + 8$

b. $2x + 6 \geq 0$

c. $\frac{-x}{2} + \frac{3}{5} \leq 0$

d. $4(x - 2) + 4(x + 1) - 6 \leq 0$

e. $3x^2 + 6x \geq \frac{6x^2}{3} + 10$

f. $13x^2 + 16 \leq 0$

3. Solve each of the following inequalities by using the transformation rules.

a. $32 - 14x \geq 20x - 8$

b. $5x + 5x + 2x \leq -24$

c. $7(x - 2) < 4x - 8$

d. $5(x - 3) < 7(x + 6)$

e. $\frac{3x + 4}{2} \geq 10$

f. $\frac{2 - 3x}{4} > x + 4$

g. $\frac{3}{4}x + \frac{2}{3} < \frac{5}{6}x + \frac{4}{5}$

h. $-5x + 7 \leq 1.4x - 17$

i. $\frac{2}{3}x + \frac{3}{4} < \frac{4}{5}x + \frac{5}{6}$

Challenge problems

Solve for x

4. $x + 0.000894 \leq -0.009764$

5. $8x - 0.00962 \leq 7x + 0.00843$

6. $x + 0.001096 \geq -0.005792$

7. $6x - 0.000834 < 5x - 0.000948$

2.2.2 Solution Set of Linear Inequalities

Activity 2.3

Find the solution set of the following inequalities under the given domain.

a. $10x+14 < 25$; (domain is $\mathbb{N}$)

b. $5(2+x) > 18+6x$; (domain is $\mathbb{W}$)

c. $2-3x \geq 10$; (domain is $\mathbb{Q}$)

d. $10-2x \leq 4x-2$: (domain is $\mathbb{Z}$.)

In this topic you will solve linear inequalities by applying the necessary rules of transformation.

- To find the solutions of a given inequality, you will use the rules of transformation for inequalities to get successive equivalent inequalities so that the least simplified form is either $x > a$ or $x < a$ or $x \leq a$ or $x \geq a$.

In solving a linear inequality of the form $ax + b > 0$, $a \neq 0$, you have to consider two cases. These are:

When $a > 0$ and when $a < 0$

Case 1: when a > 0

$ax + b > 0$.........Given inequality

$ax + b - b > 0 - b$ ......Subtracting b from both sides.

$ax > -b$............Simplifying

$\frac{ax}{a} > \frac{-b}{a}$.......... Dividing both sides by a since $a > 0$

$x > \frac{-b}{a}$ ....Simplifying

Therefore, the solution set is $\left\{x : x > \frac{-b}{a}\right\}$.

Case 2: When a < 0

$ax + b > 0$..........Given inequality

$ax + b - b > 0 - b$...........Subtracting b from both sides

$ax > -b$......Simplifying

$\frac{ax}{a} < \frac{-b}{a}$ .... Dividing both sides by a since $a < 0$

$x < \frac{-b}{a}$ .... Simplifying

Therefore, the solution set is $\left\{x : x < \frac{-b}{a}\right\}$.

Definition 2.10: The set of numbers from which value of the variable may be chosen should be meaningful and it is called the domain of the variable.

Example 23: Given the domain = {2, 4, 6, 8, 10, 12, 14}. Find the solution set of the inequality $x - 5 > 6$.

Solution:

$x - 5 + 5 > 6 + 5$

$x > 11$

Since 12 and 14 are the solution of the given inequality $x - 5 > 6$, these numbers the set containing is called the solution set of $x - 5 > 6$

You can now define the term solution set or truth set.

Definition 2.11: The set containing all the solutions of an inequality is called the solution set or truth set of the inequality and denoted by S.S or T.S.

Example 24: Find the solution set of the following inequalities under the given domain.

a. $2x + 10 < 10$: $x \in \mathbb{N}$

b. $-10x - (5 + 3x) \geq 0$; $x \in \mathbb{W}$

c. $2(x + 1) \leq 8x - (4x - 10)$; $x \in \mathbb{Q}$

d. $14(x - 4) < 8x - 16$; $x \in \mathbb{Q}^+$

Solution:

a. $2x + 10 < 10$; $x \in \mathbb{N}$……Original inequality

$2x + 10 + (-10) < 10 + (-10)$………..Subtracting 10 from both sides

$2x < 0$ …………….Simplifying

$\frac{2x}{2} < \frac{0}{2}$ ………….. Dividing both sides by 2

$x < 0$

Solution set = { }. Because there is no natural number less than zero.

b. $-10x - (5 + 3x) \geq 0$; $x \in \mathbb{W}$ ……..Original inequality.

$-10x - 5 - 3x \geq 0$…………Remove parenthesis

$-10x - 3x - 5 \geq 0$………..Collect like terms

$-13x-5 \geq 0$……………..Simplifying

$-13x - 5 + 5 \geq 0 + 5$………..Adding 5 from both sides

$-13x \geq 5$…………….Simplifying

$x \leq \frac{-5}{13}$ ………...Why?

Solution set= { }. Because there is no whole number less than or equal to $\frac{-5}{13}$.

c. $2(x + 1) \leq 8x- (4x - 10)$; $x \in \mathbb{Q}$ …….Original inequality

$2x + 2 \leq 8x - 4x + 10$……….Remove parenthesis

$2x + 4x + 2 \leq 8x - 4x + 4x + 10$………..Adding 4x from both sides

$6x + 2 \leq 8x + 10$………..Simplifying

$6x - 8x + 2 \leq 8x - 8x + 10$ .... Subtracting 8x from both sides

$-2x + 2 \leq 10$ ...... Simplifying

$-2x + 2 - 2 \leq 10 - 2$………..Subtracting 2 from both sides

$-2x \leq 8$………………..Simplifying

$\frac{-2x}{2} \geq \frac{8}{2}$ .... Dividing both sides by 2

$x \geq -4$……….Remember to reverse the sign of the inequality.

Solution set = $\{x \in \mathbb{Q}: x \geq -4\}$

d. $14 (x - 4) < 8x-16$; $x \in \mathbb{Q}^+$ .... Original inequality

$14x - 56 < 8x - 16$…………Remove parenthesis

$14x - 8x - 56 < 8x - 8x - 16$……….Subtracting 8x from both sides

$6x - 56 < -16$……….Simplifying

$6x - 56 + 56 < -16 + 56$…………..Adding 56 from both sides

$6x < 40$ ………..Simplifying

$\frac{6x}{6} < \frac{40}{6}$ ....Dividing both sides by 6

$x < \frac{20}{3}$ .... Simplifying

$$S.S = \left\{x \in \mathbb{Q}^+ : x < \frac{20}{3}\right\}$$

2.2.3. Applications of Linear Inequalities

Provides several commonly used statements to express inequalities.

Table 2.1.

English phrase	Mathematical Inequality
✓ **a is less than b**	$a < b$
✓ **a is greater than b** ✓ **a exceeds b**	$a > b$
✓ **a is less than or equal to b** ✓ **a is at most b** ✓ **a is no more than b**	$a \leq b$
✓ **a is greater than or equal to b** ✓ **a is at least b** ✓ **a is no less than b**	$a \geq b$

Example 25: Translating Expressions Involving Inequalities.

a. The speed of a car, S, was at least 220 km/hr.

b. Aster's average test score ,t, exceeded 80.

c. The height of a cave, h, was no more than 20m.

d. The temperature on the tennis court, t, was no less than 200°F.

e. The depth, d, of a certain pool was at most 10m.

Solution:

a. $s \geq 220$ c. $h \leq 20$ e. $d \leq 10$

b. $t > 80$ d. $t \geq 200°F$

Example 26: To earn grade A in a maths class, Aisha must have average score at least 90 on all of her tests. Suppose Aisha has scored 80, 86, 90, 94 and 96 on her first five maths tests. Determine the minimum score she needs on her sixth test to get an A in the class.

Solution:

Let x represent the score on the sixth test……….Lable the variable

(Average of all tests) ≥ 90………Create a verbal model.

$\dfrac{80+86+90+94+96+x}{6} \geq 90$……..The average score is found by taking the sum of the test scores and dividing by the numbers of scores.

$$\frac{446 + x}{6} \geq 90 \text{ ……………Simplify}$$

$$6\left(\frac{446 + x}{6}\right) \geq 90 \times (6) \text{ ………..Multiply both sides by 6 to eliminate the}$$

denominator fractions.

$446 + x \geq 540$……………….Solve the inequality

$446 - 446 + x \geq 540 - 446$…….Subtracting 446 from both sides

$x \geq 94$ …………..Simplifying

Aisha must score at least 94 on her sixth test to receive an A in the course.

Example 27: Eight times a natural number is increased by 4 times the number is less than 36. What are the possible value of this number?

Solution: let n be the number

$8n + 4n < 36$………….Translated inequality.

$12n < 36$……………..Collect like terms.

$$\frac{12n}{12} < \frac{36}{12} \text{ ………Dividing both sides by 12.}$$

$n < 3$ ………..Simplifying.

Therefore, the required natural number is less than 3. Thus the number is either 1 or 2.

Example 28: For the region on the right figure. Find all values of x for which the perimeter is less than 37cm (see Figure 2.7 to the right).

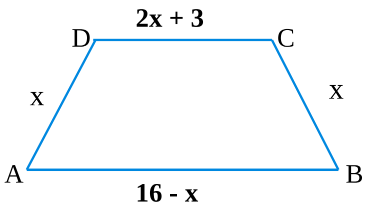


Figure 2.7

Solution: Consider the following Figure 2.8

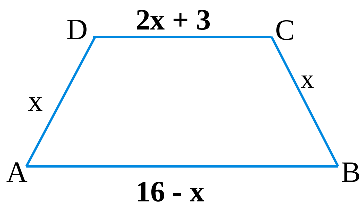


Figure 2.8

Let the perimeter = p

$AB + BC + CD + DA = P$

$16 - x + x + 2x + 3 + x = P$…….Substitution

Thus $P = 19 + 3x$, but we need $P = 19 + 3x < 37$

$19 + 3x < 37$

$3x < 37 - 19$

$3x < 18$

$x < 6$ and since x represents length, $x > 0$

Therefore, the values of x is $x < 6$ or $\{x \in \mathbb{N}: x < 6\} = \{1, 2, 3, 4, 5\}$…(Why)?

Exercise 2E

Solve each of the following word problems.

1. Twice a number x exceed 5 by at least 4. Find all possible values of x.
2. A natural number is less than the sum of its opposite and 8. Find all such numbers.
3. Find the two smallest consecutive even integers whose sum is at least 51.
4. The perimeter of a rectangle field is 118m. If the length of the rectangle is 7m less than twice the width, what is the length of the field?

Summary For Unit 2

1. **A constant (a number), a variable or product of a number and variable is called a term.**
2. **Like terms or similar terms are terms whose variables and exponents of the variables are exactly the same but only differ in the numerical coefficients.**
3. **In the product of a number and variable, the factor which is a numerical constant of a term is called** a numerical coefficient.
4. **An equation is a mathematical statement in which two quantities or two algebraic expressions are connected by the equality sign "=".**
5. **A linear equation in one variable x is an equation which can be written in standard form $ax + b = 0$, where a and b are constant numbers with $a \neq 0$.**
6. **Two equations are said to be equivalent, if and only if their solution sets are equal.**
7. **An inequality is a mathematical statements which contains the inequality symbols <, >, ≤ or ≥ to express that one quantity is greater than (or less than) another quantity.**
8. A linear inequality in one variable "x" is an inequality that can be written in the form of $ax + b < 0$, $ax + b \leq 0$ **or**

   $ax + b > 0$, $ax + b \geq 0$ **where a and b are rational numbers and** $a \neq 0$.

9. **Rules of transformation for equation:**

Let a, b and c be any rational numbers

a) If a =b, then a + c = b + c…………Addition property of equality.

b) If a = b, then a - c = b - c ……….Subtraction property of equality.

c) If a = b, then a × c = b × c………..Multiplication property of equality.

d) If a = b, then $\frac{a}{c} - \frac{b}{c}$ (provided c≠0). …..Division property of equality.

10. **Rules of transformation for inequality:**

Let a, b and c be any rational numbers

a) If a < b, then a + c < b + c………..Addition property of inequality.

b) If a < b, then a – c < b - c………….Subtraction property of inequality.

c) If c is positive and a < b, then ac < bc……Multiplication property of inequality.

d) If c is positive and a < b, then $\frac{a}{c} < \frac{b}{c}$ …….Division property of inequality.

e) If c is negative and a < b, then ac > bc ………Multiplication property of inequality.

f) If c is negative and a < b, then $\frac{a}{c} > \frac{b}{c}$ ………Division property of inequality.

Miscellaneous Exercise 2

1. Solve each of the following equations by using the rules of transformation.
   a) $-(-7x + 9) + (3x - 1) = 0$
   b) $5(3y) + 5(3 + y) = 5$
   c) $2x - \frac{1}{4} = 5$
   d) $-1.8 + 2.4x = -6.6$
   e) $\frac{6}{7} = \frac{1}{7} + \frac{5}{3}y$
   f) $5x - 3 - 4x = 13$
   g) $16y - 8 - 9y = -16$
   h) $6x-5-16x = -7$
   i) $\frac{3}{7}x - \frac{1}{4} = \frac{-4}{7}x - \frac{5}{4}$
2. Solve the equations using the steps as out lined in the text and finally check the result.
   a) $4(x + 15) = 20$
   b) $4(2y + 1) - 1 = 5$
   c) $5(4 + x) = 3(3x - 1) - 9$
   d) $6(3x - 4) + 10 = 5(x - 2) - (3x + 4)$
   e) $-5y + 2(2y + 1) = 2(5y - 1) - 7$
   f) $-2(4p + 1) - (3p - 1)) = 5(3 - p) - 9$
   g) $5 - (6y + 1) = 2((5y - 3) - (y - 2))$
   h) $7(0.4y-0.1)=5.2y+0.86$
3. Explain the difference between simplifying an expression and solving an equation.
4. Which properties of equality would you apply to solve the equation $4x + 12 = 20$?
5. Which properties of equality would you apply to solve the equation $4x - 12 = 20$?
6. The sum of two consecutive integers is -67. Find the integers?
7. The sum of the page numbers on two integers facing pages in a book is 941. What are the page numbers?

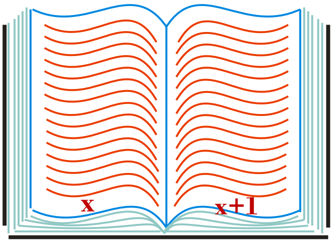


Figure 2.9

8. If y represents the smallest of three consecutive odd integer, write an expression to represent each of the next two consecutive odd integers.
9. Three consecutive odd integers are such that 3 times the smallest is 9 more than twice the largest. Find the three numbers.
10. a) Simplify the expression: $6(x + 2) – (4x - 14)$
    b) Simplify the expression: $-(10x – 1) – 4(x + 8)$
11. Solve each of the following inequalities by using the rules of transformation.
    a) $-4x – 8 \leq 22$
    b) $-14y - 6 \leq 6y$
    c) $4x + 2 < 6x + 8$
    d) $8 - 6(x - 3) > -4x + 12$
    e) $3 - 4(y - 2) > -5y + 6$
    f) $\frac{7}{6}x + \frac{4}{3} \geq \frac{11}{6}x - \frac{7}{6}$
12. Find the solution set of each of the following inequalities under the given domain.
    a) $4x - \frac{1}{3} < 6x + 4\frac{2}{3}, x \in \mathbb{W}$
    b) $9x – 4 < 13x – 7, x \in \mathbb{Z}$
    c) $0.7 (x + 3) < 0.4 (x + 3), x \in \mathbb{Q}$
    d) $3(x + 2) – (2x – 7) \leq (5x – 1) -2(x + 6), x \in \mathbb{N}$
    e) $6 – 8 (y + 3) + 5y > 5y – (2y – 5) + 13; x \in \mathbb{Q}^+$
13. Find all values of X for which the perimeter is at most 32.

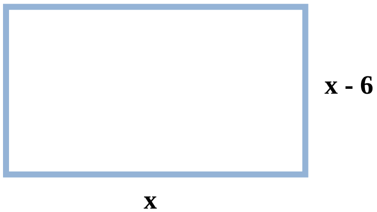


Figure 2.10

14. A board with 86 cm in length must be cut so that one piece is 20 cm longer than the other piece. Find the length of each piece.

Figure 2.11

15. Solve for X: $\frac{1}{2}x + \frac{1}{3}x - 2\left[\frac{x-2}{4}\right] \geq 2(x-1) + 3x.$

UNIT 3

RATIO, PROPORTION AND PERCENTAGE

Unit outcomes:

After completing this unit, you should be able to:

- understand the notions of ratio and proportions.
- solve problems related to percentage.
- make use of the concept of percentage to solve problems of profit, loss and simple interest.

Introduction

You may see when people are comparing two or more quantities that are measured in the same unit. Have you ever compared such quantities by yourself or with your friends? In this topic you will learn mathematical concept of comparing quantities known as ratio, proportion, percentage and the application of percentage to calculate profit, loss and interest.

3.1. Ratio and Proportion

Group work 3.1

Discuss with your friends

1. **Write a simple ratio for each of the following.**

   a) Cats to hens

Figure. 3.1 cats

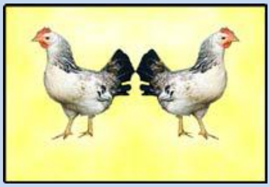


Figure. 3.2 hens

   b) Car to bicycles

Figure. 3.3 car

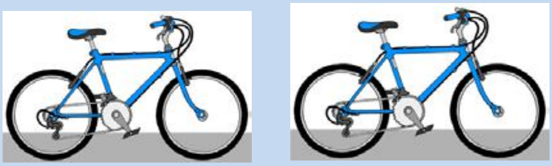


Figure. 3.4 bicycles

2. In Firure 3.5 below write the ratio of cofee to milk.

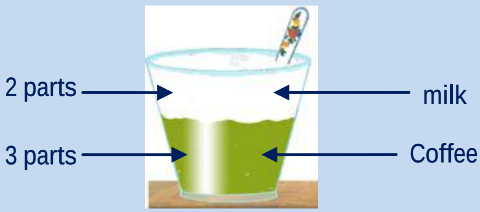


Figure. 3.5

3. Can you define a ratio based on the above activities in your own words.
4. What is the ratio of girls to boys in your class?

In this unit you will learn more about ratio, proportion and percentage. In all cases they deal with comparing quantities and give idea of their numerical relationship.

Definition 3.1: Ratio is a comparision of two or more quantities (magnitudes) of the same kind, in the same unit.

Note:

1. The ratio of one quantity a to another quantity b is usually denoted by a:b or $\frac{a}{b}$ ($b \neq 0$):
   i. a and b are called terms of the ratio.
   ii. the first number a is called antecedent and the second number b is called consequent.
2. The ratio of a to b is written as any one of the following forms:
   i. a:b (read as "the ratio of a to b or simply a is to b").
   ii. $\frac{a}{b}$ is (read as a over b (b$\neq 0$).
   iii. a$\div$ b (read as a divided by b provide b$\neq$0).
3. While comparing two quantities interms of ratio, one must bear in mind the following:
   i. the two quantities must be the same kind.
   ii. for any two quantities a and b a:b $\neq$ b:a.
   iii. ratio have no unit (it is simply a number).
   iv. the units of measurement of the two quantities must be the same.
   v. in most cases the ratio a:b is written in simplified form where a and b are natural numbers.

Example 1. Write down the ratio of the first number to the second one, in the simplest form:

a. 70 to 210 b. 48 to 244 c. 90 to 180 d. 48 to 108

Solution:

a. The ratio of 70 to 210 = $\frac{70}{210} = \frac{70\times1}{70\times3} = \frac{1}{3} = 1:3$

b. The ratio of 48 to 244 = $\frac{48}{244} = \frac{4\times12}{4\times61} = \frac{12}{61} = 12:61$

c. The ratio of 90 to 180 = $\frac{90}{180} = \frac{90\times1}{90\times2} = \frac{1}{2} = 1:2$

d. The ratio of 48 to 108 = $\frac{48}{108} = \frac{12\times4}{90\times2} = \frac{4}{9} = 4:9$

Example 2. Write down the ratio of the second number to the first one, in the simplest form:

a. 88 to 132   c. 3500 to 6500

b. 2000 to 2250   d. 1100 to 1540

Solution:

a. The ratio of 132 to 88= $\frac{132}{88} = \frac{44\times3}{44\times2} = \frac{3}{2} = 3:2$

b. The ratio of 2250 to 2000 =$\frac{2250}{2000} = \frac{225}{200} = \frac{25\times9}{25\times8} = \frac{9}{8} = 9:8$

c. The ratio of 6500 to 3500 =$\frac{6500}{3500} = \frac{65}{35} = \frac{5\times13}{5\times7} = \frac{13}{7} = 13:7$

d. The ratio of 1540 to 1100 =$\frac{1540}{1100} = \frac{154}{110} = \frac{22\times7}{22\times5} = \frac{7}{5} = 7:5$

Example 3: A mathematics class consists of 25 boys and 35 girls.

a. What is the ratio of boys to girls?

b. What is the ratio of girls to boys?

c. What is the ratio of girls to the total number of students in the class?

d. Wht is the ratio of boys to the total number of students in the class?

Solution:

a. Ratio $= \frac{\text{number of boys}}{\text{number of girls}}$

$25 : 35 = \frac{25}{35} = 5:7$

b. Ratio $= \frac{\text{number of girls}}{\text{number of boys}}$

$35:25 = \frac{35}{25} = 7:5$

c. Total number of students = number of girls + number of boys

= 35+25

= 60

Then Ratio $= \frac{\text{number of girls}}{\text{Total number of students}}$

$= \frac{35}{60} = \frac{5\times7}{5\times12} = \frac{7}{12} = 7:12$

d. Ratio = $\frac{\text{nubmer of boys}}{\text{total nubmer of sutdents}}$

$$= \frac{25}{60} = \frac{5\times5}{5\times12} = \frac{5}{12} = 5:12$$

Example 4: Divide 800 in the ratio of 3:5.

Solution: The sum of the parts =3+5=8

Note that parts are terms of the ratio.

- The first part is $\frac{3}{8}$ of 800 =$\frac{3}{8} \times 800 = 300$

  The second part is $\frac{5}{8}$ of 800=$\frac{5}{8} \times 800 = 500$

Example 5: A painter made 28 gallons of paint using white pigment, linseed oil, dryer, and turpentine in the ratio of 3:2:1:1 respectively. How many gallons of each material did she use?

Solution: The sum of the parts =3+2+1+1=7

i. The white pigment $\frac{3}{7}$ of 28=$\frac{3}{7} \times 28 = 12$ gallons.

ii. The linseed oil $\frac{2}{7}$ of 28 =$\frac{2}{7} \times 28 = 8$ gallons.

iii. The dryer $\frac{1}{7}$ of 28 =$\frac{1}{7} \times 28 = 4$ gallons.

iv. The turpentine $\frac{1}{7}$ of 28 =$\frac{1}{7} \times 28 = 4$ gallons.

Example 6: If A:B =3:6 and B:C= 6:7, then find A:C.

Solution:

i. First method

A:B =3:6 is $\frac{A}{B} = \frac{3}{6}$

B:C = 6:7 is $\frac{B}{C} = \frac{6}{7}$

This $\left(\frac{A}{B}\right) \times \left(\frac{B}{C}\right) = \frac{A}{C}$

Then $\frac{A}{C} = \frac{3}{6} \times \frac{6}{7} = \frac{3}{7}$

Therefore, $\frac{A}{C} = \frac{3}{7}$ or A:C =3:7

ii. Second method

$a.$ $\frac{A}{B} = \frac{3}{6}$ ………….Solve for B interms of A

3B = 6A ……….Cross multiplication

$\frac{3B}{3} = \frac{6A}{3}$ ... ...Dividing both sides by 3

Therefore B=2A…………Equation 1

$\frac{B}{C} = \frac{6}{7}$………Solve for B interms of C

7B= 6C…….Cross multiplication

$\frac{7B}{7} = \frac{6C}{7}$…… Dividing both sides by 7.

Therefore B=$\frac{6}{7}$C……….Equations 2

Equating equation (1) and equation (2), you will get;

2A= $\frac{6}{7}c$

14A = 6c……….Cross multiplication

$\frac{A}{C} = \frac{6}{14}$

Therefore $\frac{A}{C} = \frac{3}{7}$ or A:C =3:7

Example 7: If a,b and c are numbers such that a:b:c = 3:4:5 and b=20. Find the sum of a+b+c.

Solution: Consider the ratio;

$\frac{a}{b} = \frac{3}{4}$

Then $\frac{a}{20} = \frac{3}{4}$ ………... Substitution

4a=60………… Cross multiplication

$\frac{4a}{4} = \frac{60}{4}$……….. Dividing both sides by 4

a=15

Similarly consider the ratio;

$\frac{b}{c} = \frac{4}{5}$

Then $\frac{20}{c} = \frac{4}{5}$ …………….. Substitution

4c=100……..Cross multiplication

$\frac{4c}{4} = \frac{100}{4}$………Dividing both sides by 4

c=25

Therefore, a+b+c =15+20+25

= 60

Example 8: Find the ratio of a to b if 2a=3c and 12c =7b.

Solution:

2a=3c……….Original equation

a= $\frac{3}{2}$c ………. Solve for a

Now 12c=7b…………...Original equation

b= $\frac{12}{7}$c … … … Solve for b

Therefore, the ratio of a to b: $\frac{a}{b} = \frac{3/_{2}c}{12/_{7}c} = \frac{7}{8}$

Exercise 3A

1. Write down the ratio of the first number and the second one in the simplest form.
   a. 48 and 80
   b. 4.8and 9.6
   c. 10.2 and 13.6
   d. 13.6 and 10.2
   e. $\frac{14}{21}$ *and* $\frac{10}{15}$
   f. $2\frac{11}{12}$ *and* $1\frac{2}{3}$
2. Express the following ratios as fractions in their lowest term:
   a. 4 Birr to 16 cents
   b. 5 days to 100 hrs
   c. 3.5 kg to 6500 grams
   d. 2 Literes to 2250 millilitres
   e. 3 min 54 sec to 2 min 6 sec
   f. 6 litres to 10 c.c
3. Find two numbers whose ratio is 3 to 5 and whose sum is 88.
4. The ratio of two numbers is 7:3 and their sum is 50, find the two numbers.
5. The ratio of the measures of the angles of a triangle is 1:2:3. Find the measure of each angle.
6. Four friends contribute the sum of money to a charitable organization in the ratio 1:3:5:7. If the largest amount contributed is Birr 35. Calculate the total amount contributed by the four people.
7. If 2A:3B =5:6 and 3B:2C =36:15, then find A:C.
8. If a:b =3:2 find the value of $\frac{a+b}{a-b}$.
9. Two numbers have ratio 12:5. Their difference is 49. Find the two numbers.
10. A string of length 160cm is cut into 2 pieces, in the ratio 3:5. Find the length of each piece.

Challenge Problems

11. The ratio of two numbers is 4:5. After adding 20 to the smaller number and subtracting 20 from the greater number the ratio becomes 14:13. Find the numbers.
12. Consider $\left\{\frac{4}{7}, \frac{5}{9}, \frac{6}{11}, \frac{7}{13}, \frac{8}{15}\right\}$. What is the the largest ratio?
13. Find the ratio of the areas of the squares PQRS to that of ABCD where PQ = 9cm and AB = 5cm.

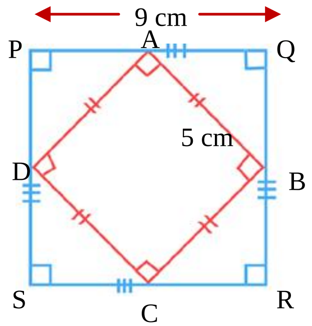


Figure. 3.6

Proportion

Group work 3.2

Discuss with your /friends/.

1. Find x, if 4.8, 6.0, x and 8.5 are in proportion.
2. Find the unknown value in the proportion: (3x+6):5=(x+8):3.
3. Can you define a proportion by your own words?

Proportion is a relationship between two quantities (or variables) in which one is a constant multiple of the other. In general, proportion can be defined as follows:

Definition 3.2: A proportion is the equality of two ratios.

Example 9: Some examples of proportion are:

a. $\frac{5}{8} = \frac{10}{16}$

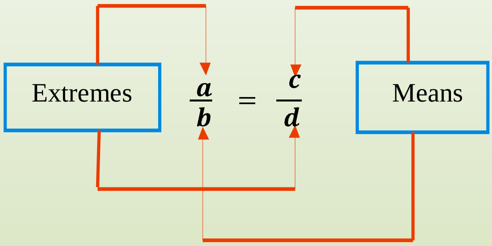
c. $\frac{1}{2} = \frac{5}{10}$

b. $\frac{2}{3} = \frac{6}{9}$

d. $\frac{3}{6} = \frac{40}{80}$

Note:

1. If a,b,c and d are four non-zero ratioal numbers such that $\frac{\mathbf{a}}{\mathbf{b}} = \frac{\mathbf{c}}{\mathbf{d}}$ then a, b, c and d are said to be terms of the proportion. Thus a proportion is an equation that shows the equality of two ratios.
2. The terms a,b,c, and d are also called the first, the second, the third and the fourth terms of the proportion respectively.
3. The proportion $\frac{a}{b} = \frac{c}{d}$ can be written as a:b = c:d. Here 'a' and 'd' are said to be the end terms (extremes), while 'b' and 'c' are said to be the middle terms (means) of the proportion.
4. In a proportion, the proudcut of the means equals the proudcut of the extremes. This product is called the cross product of a proportion. That is, if $\frac{a}{b} = \frac{c}{d}$ or a:b=c: d then a$\times$ $\mathbf{d} = \mathbf{b} \times \mathbf{c}$. The cross product can be shown diagrammatically as Shown in the adjacent figure:



5. One way to determine whether two ratios form a proportion is to find their cross products. If the cross products of the two ratios are equal, then the ratios form a proportion.

Example 10: Use cross products to determine whether each pair of ratios forms a proportion or not.

$a. \frac{32}{160} = \frac{1}{5}$ b. $\frac{5}{16} = \frac{20}{64}$

Solution:

$a. \frac{32}{160} = \frac{1}{5}$

$32\times 5 = 160 \times 1$.......Write crossproudct

160=160

Therefore, $\frac{32}{160}$ and $\frac{1}{5}$ form a poroportion because the crossproducts are equal.

b. $\frac{5}{16} = \frac{20}{64}$

$5\times 64 = 16 \times 20$………Write cross proudct

320 = 320

Therfore, $\frac{5}{16}$ and $\frac{20}{24}$ form a proportion because the cross products are equal.

Example 11: Find the unknown terms in each of the following terms.

a. 15:12 = 35: x (provided x≠ 0) b. 3:6 = x:12

Solution:

a. 15:12=35:x

Then $\frac{15}{12} = \frac{35}{x}$

15×x=12×35 ………….. Write crossproduct

15x=420 …………….. Simplifying

$\frac{15.x}{15} = \frac{420}{15}$ …………… Dividing both sides by 15

x = 28

Therefore, the value of x is 28

b. 3:6 = x:12

Then $\frac{3}{6} = \frac{x}{12}$

6× x=3 × 12………Write crossproduct

6x =36……….Simplifying

$\frac{6x}{6} = \frac{36}{6}$ ……...Dividing both sies by 6

x= 6

Therefore, the value of x is 6

Example 12: Given the proportion 3:15 =12:60 then find the sum of the means.

Solution: 3:15 =12:60………Given proportion

Then $\frac{3}{15} = \frac{12}{60}$ since 12 and 15 are means

Therefore, 12+15=27

Hence the sum of the mean of the proportion is 27.

Example 13: What number should be subtracted from each of the numbers 17, 14, 22, 18 so that the differences would be in proportion?

Solution: Let x be the required number to be subtracted from each of the given numbers.

Then, the numbers (17-x), (14-x), (22-x) and (18-x) are in proportion, that means $(17 - x):(14 - x) = (22 - x):(18 - x)$

Therefore $\frac{17-x}{14-x} = \frac{22-x}{18-x}$........Translated equation

$(17-x)\times(18-x)=(22-x)\times(14-x)$......Cross multiplication

$17(18-x)-x(18-x)=22(14-x)-x(14-x)$

$306-17x-18x+x^2=308-22x-14x+x^2$.........Remove parenthesis.

$306-35x+x^2=308-36x+x^2$............Collect like terms

$306-308-35x+36x+x^2-x^2=0$

$-2+x=0$...........Simplifying

$2 - 2 + x = 0 + 2$ .... Adding 2 to both sides

$x=2$

Exercise 3B

1. If x^2, xy, p, y^2 are in proportion, find the value of p.
2. Find the unknown terms in each of the following.

   a. $\frac{3}{4} = \frac{x}{20}$ c. $\frac{3}{7} = \frac{y}{133}$ e. $\frac{2.4}{17.5} = \frac{x}{1505}$ g. $\frac{15}{4} = \frac{x}{68}$

   b. $\frac{y}{36} = \frac{1}{2}$ d. $\frac{9}{2} = \frac{k}{84}$ f. $\frac{4}{11} = \frac{y}{132}$ h. $\frac{1.7}{8.5} = \frac{x}{467.5}$
3. Find the fourth proportional to the following:

   a. 15,12,35 b. a^2, ab, b^2
4. Determine whether the numbers 14, 21, 4 and 6 are in proportion.

Challenge Problems

5. Find the means proportional between each of the following:

   a. 20 and 45 b. 25 and 16
6. If $48x^2$, $64x^4$, x, $36x^2$ are in proportion, find the value of x.
7. Given the proportional 10:18 =35:63 then find:

   a. the sum of the means. c. the sum of the extremes.

   b. the product of the means. d. the product of the extremes.

Direct and Inverse Proportionality

From grade six mathematics lesson you have learnet about the type of proportion and their informal definition. Now in this sub-topic you revise more about direct and inverse proportionality.

A. Direct Proportionality

Group Work 3.3

1. Can you define a direct proportionality by your own words.
2. y varies directly proportion with x. if x=5 then y=12, thus find:
   a. the value of y when x= 7.5.
   b. the value of x when y=84.
3. If $y \propto x$ and when x=6; then y=24, so find the constant of proportionality.
4. $p \propto q$. If p=15 and constant of proportionality (k)=2, find q.
5. $x \propto y^3$ and when x=9 then y=3. Find the constant of proportionality.
6. $y \propto x^2$ and x=16 when y=128. Find the constant of proportionality.

7.E is directly proportional to F. If E is 24, then F is 6.
What is E when F is 24?

B. Inverse Proportionality

Group Work 3.4

1. Cany you define an inverse proporitionality by your own words.
2. x is inversely proportional to y. If x=15 then y=10. Find y when x=20.
3. The values of w is inversely proportional to m. If w=8 then m=25; what is m when w=20?
4. 150 men working in a factory produce 6000 articles in 15 working days. How long will it take,
   a. 50 men to produce the 6000 articles?
   b. 100 men to produce the 6000 articles?
5. x= a+b+4 and $a \propto y^2$; $b \propto \frac{1}{y}$. when y=2; x=18 and when y=1; x= -3. Find x when y=4.

3.2. Further on Percentage

Group work 3.5

Discus with your group member /friends/.

1. Based on in Figure 3.7 to the right sides answer the following questions:
   a) What is the decimal form of 93%?
   b) What is the fraction form of 93%?
2. Convert the percentage to decimals:
   a. 88% b. 414% c. 0.047%
3. Convert the percentage to fractions:
   a. 53% c. 7.6%
   b. $33^{1}/_{3}$ % d. 88%
4. Convert the decimal to percentage:
   a. 0.93 b. 10.82 c. 0.006 d. 0.88

Figure 3.7

Percentages are comparisions of a given quantity (or part) with the whole amount (which you call 100). Percentages are commonly used to describe **interest, sales prices, test results, inflation, change in profit** or **loss** and much more!

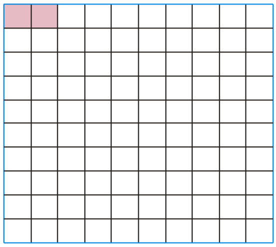
The concept of percent(%) is widely used in a variety of mathematical applications.

- ✓ A stock decreased by 22% for the year.
- ✓ The sales tax in a certain state is 7%.

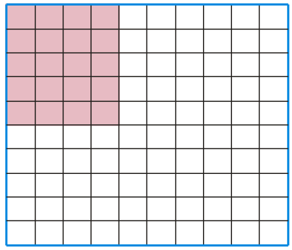
The word **percent** means per one hundred or for every haundred or hundredths. The symbol for percent is %. Hence 7%= $\frac{7}{100}$.

For example 70 percent can be written as 70% and this means the ratio of 70 to 100 or 70% = $\frac{70}{100}$.

Example 14: Look at each of the large squares below which are divided in to 100 small squares of equal size.



The shaded part is 2%



The shaded part is 20%

Figure 3.8

Activity 3.1

Discuss with your partners (friends).

1. Explain the guidelines for converting a percentage to decimal by your own words.
2. Convert the percentage to decimals.

   a.86% b. 242% c. 0.045% d. 246%
3. Explain the guidelines for converting a percentage to afractions by your own words.
4. Convert the percentage to fractions.

   a. 76% b. $83\frac{1}{3}$ % c. 226% **d. 98%**

5. Explain the guidelines for converting a decimals to a percentage, by your own words.
6. Convert the decimals to percentages.

   a. 0.135 b. 0.0035 c. 0.536 d. 0.002

7. Explain the guidelines for converting fractions to a percentage, by your own words.
8. Convert the fractions to percentages.

   a. $2\frac{5}{4}$ b. $\frac{3}{5}$ c. $\frac{7}{20}$ **d.** $\frac{1}{5}$

Exercise 3C

1. Convert each of the following percentages to decimals.

   a. 198% b. 628% c. 777% d. 0.045%

2. Express each of the following percentages as fractions.

   a. 58% b. $44\frac{1}{3}\%$ c. 7.8% d. 3.6%

3. Convert each of the decimals below to percentages.

   a. 0.96 b. 20.80 c. 0.0088 d. 28.008

4. Write each of the fractions below as percentages.

   a. $\frac{4}{15}$ b. $\frac{5}{18}$ c. $\frac{14}{25}$ d. $\frac{5}{16}$

5. Copy and complete this table 3.5 below.

Percentage	Decimal	Fraction
	0.61	
		$\frac{76}{10}$
135%		
$8\frac{1}{2}\%$		
	0.67	
		$\frac{23}{25}$
	0.089	
$76\frac{1}{4}\%$		
		$\frac{99}{5}$
96%		

Challenge Problems

6. Express the first quantity as a percentage of the second.

   a. 10m, 200m c. 45 seconds, 5 minutes

   b. 6km, 18km d. 12 hours, 1 day

7. Write the ratio of 18:45 in its simplest form and transform it into percentage.

8. Transform the ratio $5\frac{1}{8}$ into percentage.

3.2.1. Calculating Base, Amount, Percent and Percentage.

3.2.1.1. Calculating the Percentage (P)

Activity 3.2

Discuss with your friends or partner

Calculate the percentage of each of the following.

a. 60% of Birr 165
b. 90% of 250 tones
c. 96% of 32000m
d. 25% of 15.2 km
e. 4.8% of 3.6 litres
f. 6.8% of Birr 9840
g. 5% of Birr 31240
h. 10% of Birr 16.80

Percentage is part of the base number (or part of the whole) which means one number gives some part or some percent of another which denotes the whole. In short, "the part is some percent of the whole". In general, problems involving percentage are solved interms of the basic equation which is given by the formula:

Percentage = percent (Rate) x Base (whole)

Since a percent is a ratio of number of parts to 100, we can use this fact to rewrite the formula above as follows:

$$\text{Percentage} = \text{Rate} \times \text{Whole}$$

i.e P= R×B But R=r% (where r is the rate number)

Example 15: Calculate the percenatage of each of the following.

a. 80% of Birr 260
b. 9.6% of 7.2 litres
c. 90% of Birr 264
d. 34% of Birr 7000

Solution:

a. Percentage = Base × Rate

or P= B×R

$$= 260 \times \frac{80}{100}$$

$$= \frac{20800}{100} = \text{ Birr } 208$$

b. Percentage = Base × Rate

or P = B×R

$$P = \frac{9.6}{100} \times 7.2 \text{ litres}$$

= 0.6912 litres.

c. Percentage = Base × Rate

or P= B×R

$$= \text{Birr } 264 \times \frac{90}{100}$$

$$= \text{Birr } \frac{23760}{100}$$

= Birr 237.6

d. Percentage = Base× Rate

or P= B×R

$$= \text{Birr } 7000 \times \frac{34}{100}$$

$$= \frac{\text{Birr } 238{,}000}{100}$$

= Birr 2,380

3.2.1.2. Calculate the Base (B)

Activity 3.3

Discuss with your partener or friends

Calculate the base in each of the following.

a. Birr 36 is 29% of x

b. 15 cents is 5% of x

c. 16 minutes is $13\frac{1}{3}$% of time T hours

d. 90 cm is 270% of y cm.

In general, problems involving Base are solved interms of the basic equation that is given by the formula:

$$\text{Base} = \frac{\text{Percetage}}{\text{Rate}}$$

$$\text{or } B = \frac{P}{R} = \frac{P \times 100}{r}$$

Example 16: A women saves 20% of what she earns. If she saves Birr 300 a month, how much does she earn a month?

Solution: You notice that 20% is the rate (percent) and Birr 300 is the percentage, so the required quantity is the base.

$B=\frac{P\times100}{r}$............Basic formula

$B=\frac{Birr\ 300\times100}{20}$...........Substitution

B = Birr 1500

Thus, the women earns Birr 1500 with in a month.

3.2.1.3. Calculate The Rate (R)/Percent

Activity 3.4

Discuss with your teacher before starting the lesson

Calculate the rate (percent) in each of the following.

a. 400 gm to 2kg

b. birr 0.75 to birr 500

c. 30 minute to 1hr

d. 72 cm to 60m

In general, problems involving percent (rate) are solved by using the basic formula which is given below.

$$\text{Percent (Rate)} = \frac{\text{Percetage}}{\text{Base}}$$

$$\text{or } R=\frac{P}{B}$$

$$r=\frac{P}{B}\times 100\%$$

Example 17: A factory has 1200 workers of which 720 are male and the rest are female. What percent of the workers are female?

Solution: There are 1200 workers in a factory.

Thus, male workers + female workers = total workers

720+female workers =1200

female workers = 480

$r=\frac{P}{B}x100\%$.............Basic formula

$r=\frac{480}{1200}\times 100\%$..........Substitution

$r=\frac{48{,}000}{1{,}200}\%$

$r = 40\%$

Therefore, the female workers are 40% of the total workers.

Example 18: **Application Involving production**

In a certain village where farmers use two different types of fertilizers, fertilizer type A, and fertilizer type B. 10000 farmers were asked which of the fertilizers they use. It was found that 2500 farmers use type A, 4550 farmers use type B and the remaining 2950 farmers use both types A and B.

Find the percent of farmers that

a. use fertilizer type A.
b. use fertilizer type B.
c. use both types of fertilizers.

Solution: a. $r = \frac{p}{B} \times 100\%$ …….Basic formula

$r = \frac{2500}{10,000} \times 100\%$.........Substitution

$= 25\%$

Therefore, 25% use of type A.

b. $r = \frac{P}{B} \times 100\%$........Basic formula

$= \frac{4,550}{10,000} \times 100\%$.........Substitution

$= 45.5\%$

Therefore, 45.5% use of type B

c. $r = \frac{P}{B} \times 100\%$...........Basic formula

$= \frac{2,950}{10,000} \times 100\%$......Substitution

$= 29.5\%$

Therefore, 29.5% use both types A and B.

3.2.1.4. Functional Relations Among Base, Percent and Amount

In sub section 3.1 you have learnt that when parts of a ratio are multiplied by a non-zero number the ratio remains the same. If the given ratio is $\frac{a}{b}$, then $\frac{a}{b} = \frac{ka}{kb}$ where $(k \neq 0)$.

Example 19: a. $\frac{2}{56} = \frac{2\times10}{56\times10} = \frac{20}{560}$

Therefore, $\frac{2}{56} = \frac{20}{560}$

b. $\frac{28}{30} = \frac{28\times2}{30\times2} = \frac{56}{60}$

Therefore, $\frac{28}{30} = \frac{56}{60}$

If you are mainly intersted particularly in the equivalent form of fractions in which the denominators is 100, it will give us an opportunity to observe the functional relations among base, percent and amount.

Example 20: a. $\frac{9}{50} = \frac{9\times10}{10\times10} = \frac{90}{100}$

Therefore $\frac{9}{10} = \frac{90}{100}$

b. $\frac{45}{80} = \frac{45\times1.25}{80\times1.25} = \frac{56.25}{100}$

Therefore, $\frac{45}{80} = \frac{56.25}{100}$

The equivalent fractions given in example 19 above are in the form of $\frac{A}{B} = \frac{P}{100}$ where A is called **amount**, B is called **base** and P is called a **percent**.

Hence the functional relations among **base**, **percent** and **amount** can be summarized as follows:

$$\boxed{\frac{A}{B} = \frac{P}{100}}$$

Example 21: **(Income tax)** Sirak receives a salary of Birr 5,000 per month of this amount 40% is deducted for income tax. Find the amount deducted for income tax.

Solution: B= Birr 5,000

P=40

A=?

$\frac{A}{B} = \frac{P}{100}$............Basic formula

$\frac{A}{\text{Birr } 5000} = \frac{20}{100}$.......Substitution

100A= (Birr 5000)×(20)

$A = \frac{(\text{Birr } 5000)\times(20)}{100}$

A= Birr 1,000

Therefore, the amount deducted for income tax is Birr 1000.

Exercise 3D

Solve each of the following word problems.

1. In a group of tourists 34 are from U.S.A, 44 are from Japan, 64 are from Germany and 18 are from South Africa. What percentage of the group are from Japan?
2. A student scored 16 out of 25 in mathematics test. What is the student's score in percent?
3. If 25% of Tolla's salary is Birr 135.75, what is the amount of his full salary?
4. Aster sold 18 oranges. If these are 12% of her total oranges, how many oranges are not sold?
5. In woreda election where four candidates appeared for election, the winning candidate received 36,000 votes which represented 45% of the electorate. The other three candidates received 25%, 20% and 6% of votes each. How many of the electrorate voted?
6. In a class where there are a total of 80 pupils, 20 are girls and the remaining are boys. What percent of the class are boys?
7. In a class where the number of girls is 36% of the total number, there are 48 boys. How many students are there in the class?

Challenge Problems

8. The total attendance at aconcert in a theater hall was 1500, of this total 400 were children, 850 were women, and the remaning were men. Find the percent of the total attendance represented by:

   a. Children   b. Women   c. Men
9. The base is one greater than the amount. If the percentage is 93.75, then find the amount and the base.
10. Hiwot has Birr 46,000 in her account. If she plans to invest 26% of her saving in establishing a kindergarten, find the amount required.

3.3. Application of Percentage in Calculation

The concept of percentage is important for it represents a convenient way of expressing a certain types of information and it is used in solving many types of

real life problems. In this sub-section we wil easily compare quantities by means of percentage change i.e. by percenatage increase or percentage decrease.

Percentage Increase

Activity 3.5

Discuss with your friends.

1. Find the percentage change if:
   a. a quantity increases from 400 to 600.
   b. a quantity increases from 300 to 500.
2. Find the percentage change from 20 to 60.

Prices and salaries often increase by a percentage. In this section we will be finding a percentage of an amount, and then adding it to the original amount. That is: to increase a quantity by a given percentage. Find the percentage of the quantity and add it to the original quantity.

$$\text{i.e. percentage increase} = \frac{\text{actual change}}{\text{original quantity}} \times 100\%$$

Example 22: At the beginning of the year Aster had Birr 240 in her savings. At the end of the year she had managed to increase her savings to Birr 324. Calculate the percentage increase in her savings.

Solution:

$$\text{Actual change} = \text{Birr } 324 - \text{Birr } 240$$

$$= \text{Birr } 84$$

$$\text{Percentage increase} = \frac{\text{actual change}}{\text{origina quantity}} \times 100\%$$

$$= \frac{84}{240} \times 100\%$$

$$= 35\%$$

Therefore, the percentage increase is 35%.

Example 23: The students population of a school increased from 3550 in 2001 to 4620 in the year 2002. What is the rate of increase of the population between the two years?

Solution: Actual change = 4620-3550

= 1070

Percentage increase = $\frac{\text{actual change}}{\text{Origninal quantity}} \times 100\%$

$= \frac{1070}{3550} \times 100\%$

$= \frac{107000}{3550}\% = 30.14\%$

Therefore, the perceantage increase is $\frac{10700}{355}\% = 30.14\%$

Percentage Decrease

Activity 3.6

Discuss with your friends.

1. Find the percentage change if:
   a. a quantity decreases from 600 to 400.
   b. a quantity decreases from 500 to 300.
2. Find the percentage change from 80 to 30.

Reductions in numbers or prices are often expressed by using percentage. To decrease a quantity by a given percentage first find the perceantage of the quantity and then subtract it from the original quantity.

Percentage decrease = $\frac{\text{actual change}}{\text{original quantity}} \times 100\%$

Example 24: The number of student fail in chemistry test decreased from 15 to 10. What is the percenatage decrease?

Solution: Actual change = 15-10=5

Perceantage decrease = $\frac{\textit{actual change}}{\textit{original quantity}} \times 100\%$

$= \frac{5}{15} \times 100\%$

$= 33\frac{1}{3}\%$

Therefore, the percentage decrease is $33\frac{1}{3}\%$.

VAT/Value added tax/

VAT or **value added** tax is a tax imposed by the government on sales of some goods and services.

> **Note:** To apply VAT you add 15% extra on to the original cost.

Example 25: The price of a machine is Birr 3000 plus 15% VAT. How much is the VAT?

Solution: The VAT is 15 %

It is Birr $3000 \times \frac{15}{100} = \frac{45000}{100}$

= Birr 450

Therefore, the VAT is Birr 450.

Thus a person who buys this machine will pay Birr (3000 + 450) = Biir 3450 in total.

Exercise 3E

Solve each of the following word problems.

1. A man buys a washing machine whose price VAT (value added Tax) is Birr 175. Its VAT is charged at 15%, how much did the man actually pay?
2. 20% of an article is damaged and thrown away and only 20kg is left. Find its original weight.
3. A retailer agreed to take 5000 ball point pen. However he found that 12% are faulty. What was the percentage decrement?
4. Last year Abebe's salary was Birr 500. If he gets a 10% increment this year, what is his present salary.

Challenge Problems

5. A sales tax of 6% of the cost of a car was added to the purchase price of Birr 60,000. What is the total cost of the car including sales tax?

3.3.1 Calculating profit and loss as a percentage

Percenatages have a very wide application in every day transactions; one such application is the comparison of business transactions interms of percentage profit or loss.

Activity 3.7

Discuss with your teacher before starting the lesson.

1. A shop keeper buys a pair of shoes for Birr 180 and sells it for Birr 225. What is the percenatage profit?
2. A rabbit hutch costs Birr 29 plus VAT at: 15%. What is the total cost including VAT?
3. An article which cost Birr 150 was sold at a loss of 5%. What was the selling price?

When goods are bought for a sum of money and sold on a different price, there is a gain or loss on the transaction according to the selling price which is more or less than the cost price.

1. **Aprofit** is made when the selling price is greater than the cost price. The amount of profit is the difference between the selling price and the cost price.

Therefore: **Profit = Selling price(S.p)- cost price (C.P)**

To find the profit in percentage (% profit) you divide the amount of profit by the cost price and multiply by 100%.

Hence:

$$\% \text{ Profit} = \frac{\text{Selling price} - \text{cost price}}{\text{cost price}} \times 100\%$$

$$= \frac{\text{S.P} - \text{C.S}}{\text{C.S}} \times 100\%$$

$$= \frac{\text{Profit}}{\text{cost price}} \times 100\%$$

2. A **loss** is made when the cost price is greater than the selling price. The loss is the difference between the cost price and the selling price.

Therfore: **loss= Cost price (C.P) –selling price (S.P)**

To find the loss in percentage (Loss%) you divide the amount of loss by the cost price and multiply by 100%.Hence:

$$\% \text{ Loss} = \frac{\text{Cost price} - \text{selling price}}{\text{cost price}} \times 100\%$$
$$= \frac{C.P - S.P}{C.\ P} \times 100\%$$
$$= \frac{\text{Loss}}{\text{cost price}} \times 100\%$$

Example 26: A shop keeper buys a jacket for Birr 500, and gives it a clean, then sells it for Birr 640. What is the percentage profit?

Solution: Cost price (C.P) = Birr 500

Selling price (S.P)= Birr 640

Percentage profit =?

$$\% \text{ profit} = \frac{S.P - C.P}{C.P} \times 100\%$$
$$= \frac{\text{Birr } 640 - \text{Birr } 500}{\text{Birr } 500} \times 100\%$$
$$= \frac{\text{Birr } 140}{\text{Birr } 500} \times 100\%$$
$$= 28\%$$

Therefoe, the percentage profit is 28%.

Example 27 A damaged carpet which cost Birr 180 when new is sold for Birr 100. What is the percentage loss?

Solution: Cost price (C.P) = Birr 180

Selling price (S.P) = Birr 100

Percentage loss = ?

$$\% \text{ loss} = \frac{C.P - S.P}{C.P} \times 100\%$$
$$= \frac{\text{Birr } 180 - \text{Bir } 100}{\text{Birr } 180} \times 100\%$$

$$= \frac{80}{180} \times 100\%$$
$$= \frac{8000}{180}\%$$
$$= \frac{400}{9}\% \approx 44.44\%$$

Therefore, the percentage loss is 44.44%.

Example 28: An article which cost Birr 150 was sold at a loss of 5%. what was the selling price?

Solution: Cost price (C.P) = Birr 150

Percentage loss = 5%

Selling price (S.P)=?

$$\% \text{ loss} = \frac{\text{C.P}-\text{S.P}}{\text{C.P}} \times 100\%$$
$$5\% = \frac{\text{Birr } 150-\text{S.P}}{\text{Birr } 150} \times 100\%$$
$$\text{Birr } 750 = \text{Birr } 15{,}000\text{-}100 \text{ S.P}$$
$$100(\text{S.P}) = 14250 \text{ Birr}$$
$$\text{S.P} = \frac{14250}{100} \text{ Birr}$$
$$\text{S.P} = 142.50 \text{ Birr}$$

Therefore, the selling price is Birr 142.50.

Exercise 3F

Solve each of the following word problems.

1. By selling goods for Birr 175.50, a merchant made a profit of 17%. How much did the goods cost him?
2. Adealer gained a 10% profit by selling an article for Birr 330.00. what was the original price of the article?
3. A trader bought a TV set for Birr 2000 and sold it at a loss of $5\frac{1}{2}\%$. What was the selling price?
4. Girma bought 200 eggs for Birr 50 and sells them for Birr 0.30 each. Did he get profit? If so find his profit percenatage.
5. A company earned a profit of Birr 880,000 last year and Birr 970,000 this year. What is the percent change in a profit between the two years?

6. A trader bought 40 shirts for Birr 220 and sold them at Birr 27 each. Did he gain or loss percentage? Find the percentage loss or profit accordingly

Challenge Problems

7. Shop keeper M sells some goods to N and makes a profit of 15%. N resells to P at a loss of 5%. If P pays Birr 13.11 how much did M pay for the goods?
8. A profit of 24% was made when a book was sold for Birr 34.10, find the selling price that would have given a profit of 28%.

3.3.2 Simple Interest

Group work 3.6

Discuss with your friends/partners/.

1. Find the simple interest on
   a. Birr 300 for 3 years at 5%.
   b. Birr 525.00 for 4 years at $3\frac{1}{2}$%.
   c. Birr 750 for 3 years and 4 moths at $\frac{21}{2}$%.
   d. 750 dollars for $\frac{21}{2}$ years at 3%.
2. If the simple interest on a sum of money invested at $3\frac{1}{2}$% per annum for 44 years is Birr 420, find the principal.

When money is lent, particularly for business, the borrower is expected to pay for the use of the money. Charge the amount of money borrowed is called the **principal** and the charge made for the use of the money is called **interest**.

Interest on money borrowed is paid at definite time intervals **(monthly, quarterly, half-yearly or yearly).** It is usually reckoned as a **percentage** of the principal for the period stated until the loan is repaid.

The inerest paid on the original prinicipal only during the whole interest periods is called **simple interest**. Simple interest can be expessed interms of the basic interest formula as follows:

Interest = Principal × Rate × Time

that is I= PRT where I= amount of intrest

P= principal

R= The interest rate per period (Expressed as percentage)

T = time (in years)

Example 29: If Birr 1200 is invested at 10% simple interest per annum, then what is the amount after 5 years?

Solution: P= Birr 1200

R= 10%

t= 5 years

A mount (A) =?

I = PRT………Given formula

$= \text{Birr } 1200 \times \frac{10}{100} \times 5$

= Birr 600

Thus, Amount = princpale + interest

= Birr 600+Birr 1200

= Birr 1800

Example 30: How long will it take Birr 300 to double itself if it is invested at the rate of 5% simple interest per annum?

Solution: I = PRT………Given formula

A= P+I

A= P+PRT

A= P(1+RT)

Birr 600 = 300 Birr (1+0.05T)

2= 1+0.05T

$T= \frac{1}{0.05}$

T= 20 year.

Example 31: Find the simple interest on Birr 700 at 12% rate for 3 months.

Solution: P = Birr 700

R= 12%

T = 3 months = $\frac{1}{4}$ year

I = ?

I= PRT

$= \text{Birr } 700 \times \frac{12}{100} \times \frac{1}{4}$

$= \frac{\text{Birr } 8400}{400}$

= Birr 21

Therefore, the simple interest is Birr 21.

Example 32: The simple interest for nine months at 8% is Birr 37.50. Find the principal.

Solution: T= 9 moths = 0.75 year

R= 8%

I= Birr 37.50

P = ?

I = PRT …….Given formula

$P = \frac{I}{RT}$

$P = \frac{Birr\ 37.50}{8 \times 0.75} \times 100$

$P = \frac{\text{Birr } 3750}{6}$

P= Birr 625

Therefore, The principal is Birr 625.

Example 33: If Birr 80 is earned on Birr 1600 in six month, at what rate of simple interest has the interest been earned?

Solution: I= Birr 80

P = Birr 1600

T= 6 month = 0.5 year

R =?

I= PRT ……..Given formula

$R = \frac{I \times 100}{PT}$

$R = \frac{\text{Birr } 80 \times 100}{\text{Birr } 1600 \times 0.5} = 10\%$

Therefoe, the rate of the simple inerest is 10%.

Exercise 3G

Solve each of the following word problems

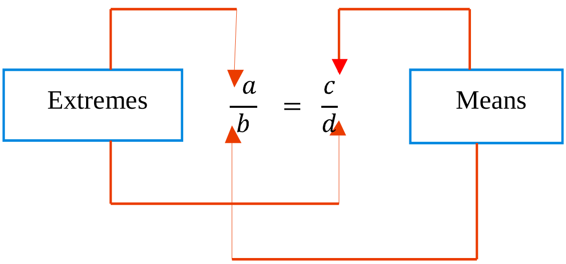
1. Find the rate percent per annum at which Birr 142 will earn Birr 59.65 in 12 years.
2. If Birr 1200 is invested at 10% simple interst per annum, then what is the amount after 5 years?
3. Find the simple interest on Birr 126.42 for 6 years at $3\frac{1}{2}$% per annum.
4. Find the time in which Birr 168.40 will earn Birr 29.47 at 5% per annum.
5. Find the prinicipal which earns Birr 115.38 in 8 years at $4\frac{1}{2}$% per annum.
6. Find the principal which amounts to Birr 142.83 in 5years at 3% per annum.
7. Find the rate percent per annum at which Birr 380 earns birr 128.25 in 7 years and 6 moths.

Challenge Problems

8. Over what period of time will be Birr 500 amount to Birr 900 at the rate of 8% simple interest.
9. A man borrows Birr 800 for 2 years at a simple interest rate of 20%. What is the total amount that must be repaid.

Summary For Unit 3

1. **Ratio is a comparision of two or more quantities/magnitudes/ of the same kind, in the same unit.**
2. **A proportion is the equality of two ratios. In a proportion, the product of the means equals the product of the extremes. This product is called the cross product of a proportion. That is, if $\frac{a}{b} = \frac{c}{d}$ or ad= bc. The cross product can be shown diagramatically as follows:**



3. y is said to be directly proportional to x(written as $y \propto x$) if there is a constant k, such that y=kx, k is called **the constant of proportionality**.
4. y is said to be in be inversely proportional to x (written $y\alpha\frac{1}{x}$) if there is a constant k such that $y=k.\frac{1}{x}$ or $k = yx$, where k is the constant of proportionality.
5. Calculating Base, amount, percent and percentage

$$\Rightarrow \text{percentage} = \frac{\text{percent}}{100} \times \text{Base}$$

$$= \text{Rate} \times \text{Whole}$$

$$\Rightarrow \text{Base} = \frac{\text{Percentage}}{\text{Rate}}$$

$$\Rightarrow \text{percent (Rate)} = \frac{\text{Percemtage}}{\text{Base}}$$

6. The functional relations among base (B), percent (P) and amount (A) can be summarized as:

$$\frac{A}{B} = \frac{P}{100}$$

7. To incease a quantity by a percentage first find the percentage of the quantity and then add it the original quantity.
8. To decrease a quantity by a percentage find the percentage of the quantity and subtract it from the original quantity.
9. Percentage increase $=\frac{\text{actual change}}{\text{original quantity}} \times 100\%$.
10. Percentage decrease $=\frac{\text{actual change}}{\text{original quantity}} \times 100\%$.
11. Percentage profit = $\frac{\text{profit}}{\text{cost price}} \times 100\%$.
12. Percentage loss = $\frac{\text{loss}}{\text{cost price}} \times 100\%$.
13. Simple interest over several years is calculated by assuming that the sum of money invested remains the same over those years and that the percentage rate of interest remains the same over those years.

    The formula for simple interest I is:

$$I = PRT$$

    Where R is the rate of interest (% per annum)

    T is the time (in years)

    P is the principal (sum of money lent or borrowed).

Miscellaneous Exercise 3

I. Write true for the correct statements and false for the incorrect ones.

1. Ratio can be defined as "for every hundred terms".
2. 25:50 = 4x:8 if x=2.
3. The ratio of 5 days to 1000 hrs is 3:25.
4. Profit % = $\frac{\text{profit}}{\text{cost price}} \times 100\%$.
5. If $\frac{x}{y} = \frac{3}{5}$, then the value of the expression $\frac{6y+5x}{3y-2x}$ is 5.
6. Percentage loss = $\frac{\text{loss}}{\text{selling price}} \times 100\%$.

II. Choose the correct answer from the given four alternaives.

7. In a school, 58% of the total numbers of students are boys. If the number of girls is 840, how many students are there in the class?
   a. 2000 b. 1800 c. 1500 d. 2100
8. Birr 300 is invested at 6% simple interest per annum. How long will it take for the interest to amonut Birr 180?
   a. 12 years b. 10 years c. 8 years d. 24 years
9. The decimal form of 672.937% is
   a. 6.72937 b. 67.2937 c. 672.937 d. 6729.37
10. The simple interest on Birr 400 invested for 4 months was Birr 12. What is the annual (yearly) rate of interest?
    a. 36% b. 4.8 % c. 7% d. 9%
11. If a,b and c are numbers such that a:b:c =2:3:5 and b= 30 then, find the sum of a+b+c?
    a. 50 b. 60 c. 90 d. 100
12. If A:B = 7:8 an B:C =12:7,find A:C in its simplest form.
    a. 2:3 b. 3:2 c. 19:6 d. 4:5
13. Three members d, m, n are in the ratio 3:6:4. Find the value of $\frac{4d-m}{m+2n}$.
    a. $\frac{7}{3}$ b. $\frac{3}{7}$ c. $\frac{2}{3}$ d. $\frac{4}{3}$
14. The ratio of the number of pigs to the anumber of horse in a farm is 2 to 3. If there are 24 horses, what is the number of pigs?
    a. 72 b. 48 c. 36 d. 16
15. (x+4), (x+12), (x-1) and (x+5) are in proportion. Find the value of x.
    a. 15 b. 12 c. 16 d.. 20

16. If (2x+3y)∝ (x+5y) or x ∝ y, then which of the follwong is true?

a. $\frac{x}{y} = \left(\frac{5k-3}{2-k}\right)$ c. x= y $\left(\frac{5k-3}{2-k}\right)$

b. x:y = (5k-3):2-k d. all are correct

17. Y is inversely proportional to cube root of x. what x= $\frac{1}{8}$ y = 2. Find the constant of proportional.

a. $\frac{1}{4}$ b. $\frac{1}{2}$ c. 1 d. 4

18. If 95% of a is equal to b, then which of the following is ture?

a. 95 a=b b. a<b c. a = b d. a>b

19. which one of the following is not equal to the rest?

a. 2% of 150 b. $\frac{3}{4}\%\ of\ 400$ c. 5% of 60 d. 6% of 50

III. Work out problems

20. If a:b:c = 5:2:3, evaluate

a. a-2b:3b-c b. a+b+c: 5a c. a-b: b+c

21. A man's income is increased in the ratio 47:40. Find the increase percentage.

22. If h:k = 2:5, x:y =3:4 and 2h+x:k+2y = 1:2, find the ratio h-x: k-y.

23. In a school the number of girls exceeds the number of boys by 15%. Find the ratio of the numbers of boys to girls.

24. In a given class room, the numer of girls is ten greater than the number of boys. If the ratio of the number of girls to the number of boys is 7:5 then find

a. the number of girls

b. the number of boys

c. the total number of students

25. A factory has 1200 workers of which 720 are male and the rest are female. What percent of the workers are female?

26. A trader bought aTV set for Birr 2000 and sold it at loss of $5\frac{1}{2}\%$, what was the selling price?

27. A merchant gains 15% by selling an article for Birr 150. By how much does he sell it to double his profit?

UNIT 4

DATA HANDLING

Unit outcomes:

After completing this unit, you should be able to:

- collect data and construct simple line graphs and pie charts
  for a given data.
- calculate the mean, median and mode of a given data.
- find the rage of a given data.

Introduction

Collection of data form a group of things help us to understand more about these things in the group. To do this the collected data should be presented systematically or pictorially so as to analysis them. In this unit you will learn how to collect simple data and present them pictorially and do some calculation on them to study their nature or property.

4.1. Collecting Data Using Tally Marks

Group Work 4.1

Discuss with your friends

Put a tick (√) in the box for the tasks that you can perform.

1. What age are you?

   Under 10 ☐

   11 – 14 ☐

   15 – 18 ☐

   Above 18 ☐

2. Which of these fruits do you like?

   Orange ☐

   Banana ☐

   Mango ☐

   Avocados ☐

3. Do you have a mobile phone?

   Yes ☐ No ☐

Figure 4.1 Mobile

Ways of collecting data

You can collect data:

- ✓ by using a questionnaire.
- ✓ by making observations and recording the results.
- ✓ by carrying out an experiment.
- ✓ from records or data base
- ✓ from the internet.

You must be careful how and when you collect data. If you want to find out what people think about marriage, for example, it is not sensible just to ask people at a wedding. They are interested in marriage and you might be led to the wrong conclusions!

Designing questions to collect data

When you are writing questions for a questionnaire:

- ✓ be clear what you want to find out, and what data you need.
- ✓ ask short, simple questions.
- ✓ Provide tick boxes with possible answers.

Note: Avoid questions which are too vague, too personal, or which may influence the answer.

Exercise 4A

Put a tick (√) in the box for the tasks that you can perform.

1. Here are same questions that are not suitable for a questionnaire. For each one, say why and write a more suitable question.

   a. Do you agree that Ethiopia should have a monarchy?

   Yes ☐ No ☐ Don't know ☐

   b. What was the weather like on your holiday?

   Terrible ☐ Quite good ☐ ok ☐

   c. Most people approve of corporal punishment. Do you?

   Yes ☐ No ☐

   d. Do you still play foot ball?

   Yes ☐ No ☐

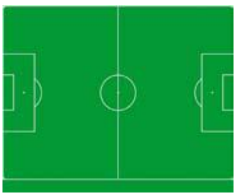


Figure 4.2 Foot ball field

e. How many hours of television do you watch?

1 ☐ 2 ☐ 3 ☐

Figure 4.3 Television

Challenge Problem

2. Use any source to find the following information. Give two reliable sources for each answer.
   a. The heights of the five highest mountains in the world.
   b. The average life expectancy of people in Ethiopia.

Some ways of presenting data

In a survey, 60 pupils were asked how many of their friends they got last Saturday. Here are the results:

3	2	4	7	7	5	8	6	7	6	7	6	8	5	4
6	8	5	3	6	7	8	8	1	7	8	6	4	8	7
6	5	7	6	9	7	6	5	8	3	7	9	4	5	7
4	3	7	7	8	5	4	7	9	6	2	5	5	6	9

Table 4.1 to see this information more clearly you can draw up a tally chart;

Number of friends	Tally	Frequency (№ survey)
0	—	0
1	|	1
2	||	2
3	||||	4
4	~~||||~~ |	6
5	~~||||~~ ||||	9
6	~~||||~~ ~~||||~~ |	11
7	~~||||~~ ~~||||~~ ||||	14
8	~~||||~~ ||||	9
9	||||	4

Note: This tally chart, or frequency table, shows the frequencies of the different numbers of friends (how often each number occurred).
Tally marks are grouped in five to make them easier to count:
𝍸 𝍸 𝍸 ||| is easier to count than |||||| ||||||||| |||
remember 𝍸 represents 5 members of the group (sample).

Example 1: Consider the following data collected from the scores of 40 sample students in a mathematics examination:

Score of 40 students on a mathematics examination

56	78	62	37	54	39	62	60
28	82	38	72	62	44	54	42
42	55	57	65	68	47	42	56
56	56	55	66	42	52	48	48
47	41	50	52	47	48	53	68

To show this information more clearly you can draw a tally chart:

Table 4.2 the tally chart for the given information is as follows:

Solution:

Scores	Tally	№ of students (Frequency)
28	|	1
37	|	1
38	|	1
39	|	1
41	|	1
42	||||	4
44	|	1
47	|||	3
48	|||	3

50	|	1
52	||	2
53	|	1
54	||	2
55	||	2
56	||||	4
57	|	1
60	|	1
62	|||	3
65	|	1
66	|	1
68	||	2
72	|	1
78	|	1
82	|	1

Example 2: A survey of 45 families was made to know about the number of children in each family. The information obtained was as follows:

2	0	3	2	2	4	4	2	2
3	3	2	3	1	2	3	3	1
2	2	4	1	2	1	1	2	1
0	3	2	3	1	5	2	2	2
1	2	3	4	2	2	2	3	2

Show this information more clearly you can draw up a tally chart.

Solution: In Table 4.3 the tally chart of the given information is as follows:

№ of children in a family	Tally marks	Frequency
0	||	2
1	卌 |||	8
2	卌 卌 卌 卌	20
3	卌 卌	10
4	||||	4
5	|	1
Total		45

Exercise 4B

1. The ages of students in a class were recorded as follows:

14	15	14	16	14	13	15	14	16	14	15	14
14	15	17	15	14	16	16	13	14	15	14	14
16	13	15	16	14	14	17	13	14	15	16	14
15	13	15	16	14	17	15	16	14	17	13	14

Show this information more clearly by drawing a tally chart.

2. For each of the following sets of data recorded at a certain Ethiopian weather station, display the information in a tally chart.

a. Hours of sunshine

5	6	0	1	3	1	4	7	5	6	6	2
4	3	1	0	7	10	9	11	5	4	7	6
9	9	11	12	12	7	9	10	11	10	9	7
8	4	6	5	7	8	10	8	6	3	6	8
3	3	4	1	10	9	11	7	2	6	10	7

b. Maximum temperature in degrees Celsius

18	19	19	21	19	21	18	18	19	18	16	18
17	18	18	17	19	18	17	16	21	22	21	21
20	22	22	23	21	18	23	21	21	22	22	17
19	17	19	21	19	19	17	19	19	16	19	17
20	22	21	20	23	21	21	22	21	21	20	20

Challenge Problem

3. Collect data for the number of exercise books that the students in your class room have. Show this information by a tally chart.

4.2. Construction and Interpretation of Line Graphs and Pie charts.

4.2.1.Line graphs

Activity 4.1

Discuss with your friends in the class.

1. Measure the hand-span of each person in your class.
2. Record the data in a tally chart.
3. Draw a line graph to display your data.

Figure 4.4 Hand span

The **line graph** is most commonly used to represent two related facts. To plot a line graph, you can take two lines at right angles to each other. These lines are called **the axes of reference**. Their intersection is called **the origin**. The number of units represented by a unit length along an axis is called a **scale**. A line graph is drawn based on pairs of measurements of two quantities. Each pair of coordinates is represented by a dot, and consecutive dots are connected by a straight line or a smooth curve. So remember the following important points in making a line graph.

1. Draw the horizontal and vertical lines (axes) and label them by using appropriate scale so that it should be enough to represent the data to be used.
2. Make a table of data arranged in pairs. The first number of each pair is read from the horizontal scale (axis) and the other number is from the vertical scale (axis). Use these numbers to locate points on the graph.
3. Connect the points by a straight line or a smooth curve.

Example 3: A car uses 1 liters of petrol for every 10 km it travels.

a. Copy and complete the Table 4.4 showing how much petrol the car uses.

Distance travelled in km	0	10	20	30	40	50	60
Petrol used in liters	0	1	2	3			

b. Draw a graph from the information in your table.

c. Work out how much petrol is used to travel 7km.

d. Work out how many kilometers had been travelled by the time 10 liters of petrol had been used.

Solution:

- 10km = 1 Liters

  40km = x

10km × x = 40km × 1iters

$$x = \frac{40\text{km} \times 1 \text{ Liters}}{10\text{km}}$$

x = 4 liters

- 10km = 1 liters

  50km = x

10km × x = 50 km × 1 Liters

$$x = \frac{50\text{km} \times 1 \text{ Liters}}{10\text{km}}$$

x = 5 Liters

- 10km = 1 liters

  60km = x

10km × x = 60 km × 1 liters

$$x = \frac{60\text{km} \times 1 \text{ litres}}{10\text{km}}$$

x = 6 Liters

a.

Distance travelled in km	0	10	20	30	40	50	60
Petrol used in liters	0	1	2	3	4	5	6

b. Plot the points (0, 0), (10, 1), (20, 2), (30, 3), (40, 4), and (50, 5) (60, 6) to draw a graph.

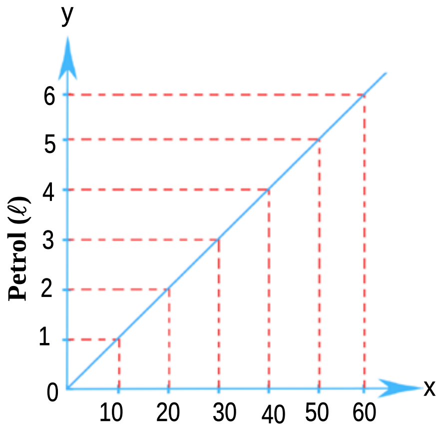


Figure 4.5

c. 10km = 1 liters

7km = x

10km × x = 7km × 1litres

$$x = \frac{7\text{km} \times 1 \text{ liters}}{10\text{km}} = \frac{7\text{km} \times 1\,\text{lit}}{10\text{km}}$$

$$x = \frac{7}{10}\text{liters}$$

x = 0.7 liters

d. 10km = 1 liters

x = 10 Liters

1 Liters × x = 10km × 10 Liters

$$x = \frac{10\text{km} \times 10\,\text{literts}}{\text{liters}}$$

x = 100 km

x = 100 kilometers

Example 4: The depth of a water in a reservoir is 144m. During a dry period the water level falls by 4m each week.

a. Copy and complete in table 4.5 showing the expected depth of water in the reservoir.

Weeks	0	1	2	3	4	5	6	7	8
Expected depth of water in m	144	140							

b. Draw a graph from the information in your completed table above.

c. How deep would you expect the level of the water to be after 10 weeks.
   If the water level falls to 96m the water company will divert water from another reservoir.

d. After how long will the water company divert water?

Solution:

a.

Weeks	0	1	2	3	4	5	6	7	8	9	10	11	12
Expected depth of water in m	144	140	136	132	128	124	120	116	112	108	104	100	96

b.

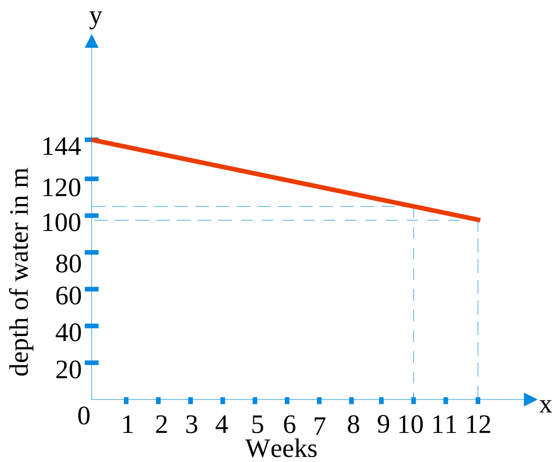


Figure 4.6

c. 104 m   d. 12 weeks

Exercise 4C

1. In Table 4.6 below gives some approximate conversion between inches and centimeters. (Hint 1 inches = 2.54 centimeters).

Centimeters	2.5	5	10	30	50
Inches	1	2	4	12	20

a. Draw a conversion graph from inches to centimeters.

b. Use your graph to find the number of centimeters in
   i. 6 inches   ii. 10 inches

c. Use your graph to find out the number of inches in
   i. 25 cm   ii. 40 cm

2. The amount of petrol (in liters) in the storage tank at a garage was measured every hour between 7am and 7pm in one day. This is the shape of the line graph showing the results:

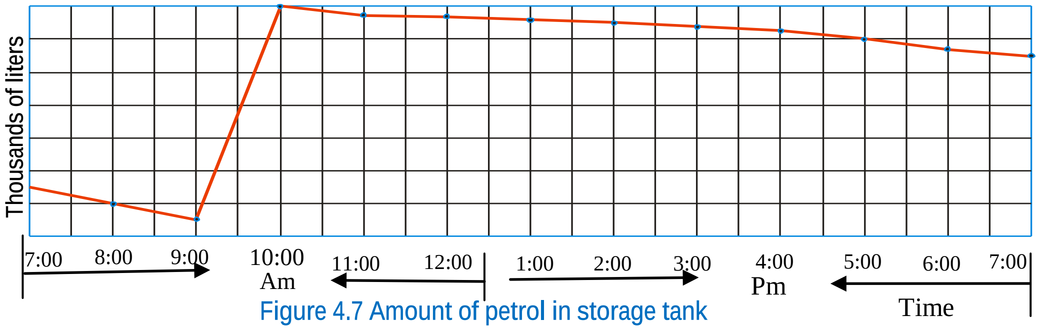


Figure 4.7 Amount of petrol in storage tank

a. When was the amount of petrol in the tank at its lowest?
b. What happened to the amount of petro between 9am and 10am
c. What can you say about the sales like between 1 pm and 4pm?
d. Give a reason for your answer to part (c).

3. Draw line graphs to represent each of the following sets of data.

a. The number of letters delivered to an office in one week (See Table 4.7)

Weeks	Sat	Sun	Mon	Tue	Wed	Thu	Fri
Letters	20	0	12	25	15	19	23

b. The temperature in Addis Ababa at midday during the first week in July (See Table 4.8)

Day	Sat	Sun	Mon	Tue	Wed	Thu	Fri
Temperature(°C)	12	16	14	11	12	15	13

4.2.2 Pie Charts

Activity 4.2

Discuss with your parents (Friends)

1. Look at in Table 4.9 below:

Age	0 – 14	15 – 25	25 – 59	60 and above
Total pupil	48%	14%	30%	8%

Draw a pie chart to display your findings.

2. Draw a pie chart whose angles at the centre are: 108°, 90°, 72°, 60° and 30°.

- **Pie chart** is a very common and accurate way of representing data specially useful for showing the relations of one item with another and one item with the whole items.

  The portion of a circular region enclosed between two radii and part of the circumference (an arc) is called **a sector of the circle.**

  The size of the sector is determined by the size of the angle formed by the two radii. The larger the angel is, the wider the sector will be.

  In a pie chart the total data is represented by the circular region as a whole and the individual data by sectors of the circle. The angle at the center of the circle is divided proportionally to determine the size of the individual data accordingly.

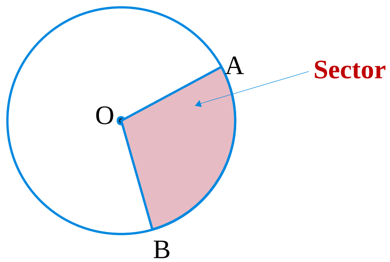


Figure 4.8 Pie chart

The following important point aid in drawing pie chart

1. Draw a circle large enough to make a clear drawing of the facts to be pictured.
2. First express the number of facts to be graphed as percentage and arrange them. Second you know that a circle has 360°. So if you divide 360° by 100 or 360 ÷ 100 of the data you will get 3.6°. This gives us the idea that 1% can be represented by 3.6°.
   Thus, multiply the percentage by 3.6° to get the size of the central angle of the sectors that represents the required data.
3. Mark off sectors of the circle corresponding to the required degrees using a protractor and draw the central angle which form the sectors.
4. A good graph should contain the following points:
   a) Have a title
   b) Be well proportioned
   c) Have scales clearly marked and labeled
   d) Different colors which shade each data
   e) Show source of the facts that it represents.

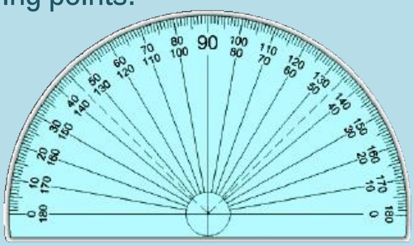


Figure 4.9 Protractor

Note: A pie charts are also called a circle graphs.

Example 5: The expenditure on different budget title of a family in amonth is given below (See Table 4.10)

Budget	Food	Education	clothing	House rent	Other	Savings	Total
Expenditures (Birr)	1200	540	900	400	360	200	3600

Show the data by a pie chart.

Solution

The measure of the angle of the sector representing the expenditure is given by the following formula:

$$\frac{\text{Expenditure on the given budget}}{\text{Total expenditure}} = \frac{\text{measure of the arc}(\theta)\text{of the sector}}{360^{o}}$$

Therefore, measure of the arc (θ) of the sector

$$= \frac{\text{Expenditure on the given budget}}{\text{total expenditure}} \times 360^{o}$$

In Table 4.11 showing the expenditure on each budget and the measure of the angle of the corresponding sector is given below:

Budget	Expenditures (Birr)	Measure of the angle($\boldsymbol{\theta}$)
Food	1200	$\frac{1200}{3600} \times 360^{o} = 120^{o}$
Education	540	$\frac{540}{3600} \times 360^{o} = 54^{o}$
Clothing	900	$\frac{900}{3600} \times 360^{o} = 90^{o}$
House rent	400	$\frac{400}{3600} \times 360^{o} = 40^{o}$
Other	360	$\frac{360}{3600} \times 360^{o} = 36^{o}$
Savings	200	$\frac{200}{3600} \times 360^{o} = 20^{o}$
Total	3600	360^{o}

On the basis of the given Table 4.11 the required pie chart is drawn below.

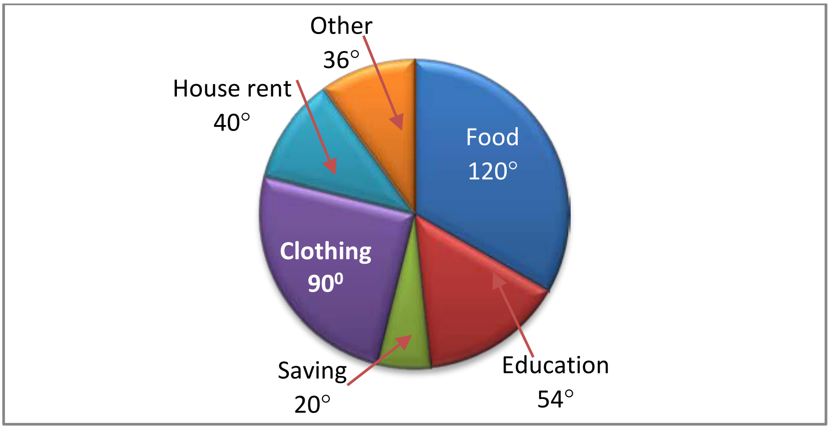


Figure 4.10 Pie chart

Example 6: The percentage of expenditure for the development programme on different budget title of a state is as show below (See Table 4.12).

budget	Agriculture	Irrigation	Electricity	Industry	Communication	Other
Percentage	25	15	15	30	10	5

Stage the data by pie chart

Solution: The measure of the angle of a sector representing expenditure on one budget title is given by the following formula:

$$\text{Measure of the angle of the sector} = \frac{\text{Percentage expenditure on each budget}}{100} \times 360^{\circ}$$

Using the above formula and finding the measure of the angle of the sector corresponding to each title of the percentage of expenditure, we will get the following table:

Budget	Percentage	Measure of the angle
Agriculture	25	$\frac{25}{100} \times 360^\circ = 90^\circ$
Irrigation	15	$\frac{15}{100} \times 360^\circ = 54^\circ$
Electricity	15	$\frac{15}{100} \times 360^\circ = 54^\circ$
Industry	30	$\frac{30}{100} \times 360^\circ = 108^\circ$
Communication	10	$\frac{10}{100} \times 360^\circ = 36^\circ$
Other	5	$\frac{5}{100} \times 360^\circ = 18^\circ$
Total	100	360°

On the basis of the given table the required pie chart is drawn in Figure 4.11below:

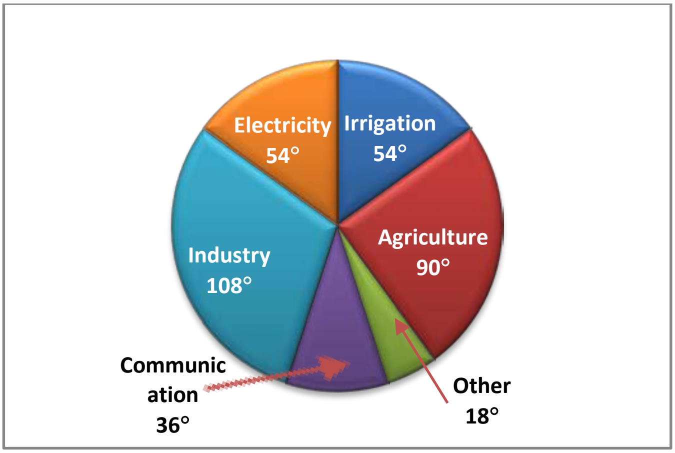


Figure 4.11 Pie chart

Example 7: The pie chart given in Figure 4.12 shows Ato Abebe's expenses and saving for the last month.

If this monthly income was Birr 1500 then find:

a. his food expenses.
b. his house rent.
c. his fuel expense.
d. his saving.

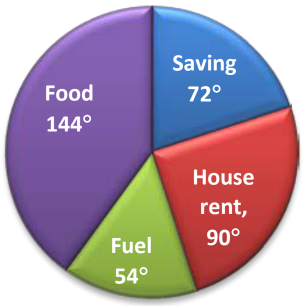


Figure 4.12 Pie chart

Solution:

Measure of the arc$(\theta)$of the sector $= \dfrac{\text{Food expense}}{\text{total amount monthly income}} \times 360^o$

a. Food expense $= \dfrac{\text{measure of the arc}(\theta)\text{of the sector} \times \text{total amount of monthly income}}{360^o}$

$= \dfrac{144^o \times 1500}{360^o}$

$= 600$

Therefore, Abebe's food expense is Birr 600

b. House rent $= \dfrac{\text{Measure of the arc }(\theta)\text{of the sector} \times \text{total monthly in come}}{360^o}$

$= \dfrac{90^o \times 1500}{360^o}$

$= 375$

Therefore, Abebe's House rent expense is Birr 375.

c. Fuel $= \dfrac{\text{Measure of the arc}(\theta)\text{of the sector} \times \text{total monthly income}}{360^o}$

$= \dfrac{54^o \times 1500}{360^o}$

$= 225$

Therefore, Abebe's Fuel expense is Birr 225.

d. Saving $= \dfrac{\text{measure of the arc}(\theta)\text{of the sector} \times \text{total monthly income}}{360^o}$

$= \dfrac{72^o \times 1500}{360^o} = 300$

Therefore, Abebe's saving is Birr 300.

Exercise 4D

1. Thirty students were asked to name their favorite chewing gum. The results are shown in Figure 4.13 below.
   a. What does the whole circle represent?
   b. Which chewing gum does the largest sector represent?
   c. What does the smallest sector represent?
   d. Use the given angles to calculate the number of students who liked strawberry chewing gum

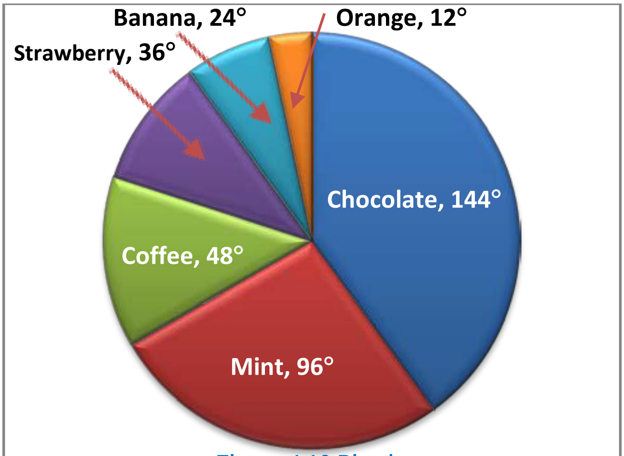


Figure 4.13 Pie chart

2. 720 students were asked how they travelled to school. The pie chart shows the results of this survey; Find
   a. how many of the students travelled to school by bus.
   b. how many students travelled on foot.

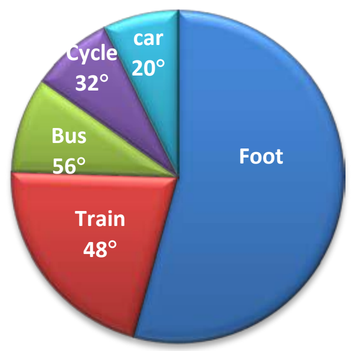


Figure 4.14 Pie chart

3. The following pie chart shows a family budget based on a net income of Birr 2400 per month.
   a. Determine the amount spent on rent.

b. Determine the amount spent on car payments.

c. Determine the amount spent on utilities.

d. How much more money is spent than saving?

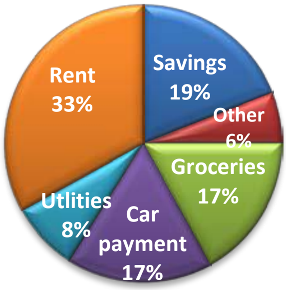


Figure 4.15 Pie chart

4. W/ro Eleni's family had an income of Birr 12,000 a year. The following pie chart shows the family used the money. W/ro Eleni's family expenditure:

How much money did the family spend on:

a. food ? d. rent?

b. savings ? e. entertainment ?

c. travel?

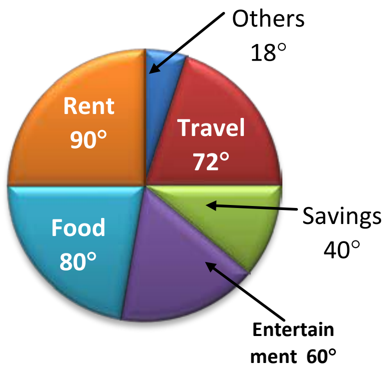


Figure 4.16 Pie chart

5. The budge for social development programme of a district is given as follows (See Table 4.13)

Item	Amount(Birr)
Education	75,000
Public health	20,000
Community development	5,000

Construct a circle graph or pie chart representing this information.

Challenge Problems

6. The total expenditure of a region council is Birr 36,000,000. The pie chart below shows how the money was spent. How much money was spent on high ways? (See Figure 4.17 below).

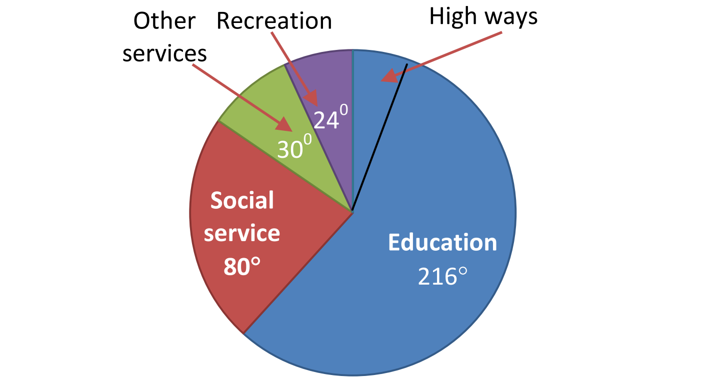


Figure 4.17 Pie chart

7. The pie chart shown below is the number of students in a certain school. There are 1200 students in the school.
   What is the number of students in grade 8?

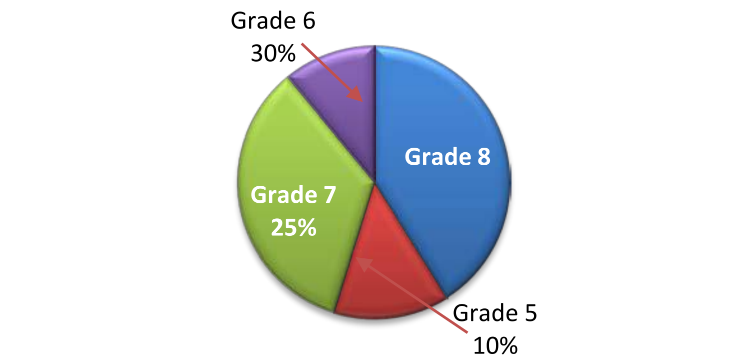


Fig 4.18 Pie chart

4.3. The Mean, Mode, Median and Range of Data

In this sub-topic you will learn about the three basic **measures of central tendency**: the **mean, median** and **mode**; while the **range** is called **measure of dispersion.**

4.3.1 The mean

Group Work 4.2

Discuss with your friends.

1. Find the mean of these numbers.

a. 132	148	141	136	134	129
b. 146	132	137	118	150	141

2. The mean of 15, 17, x, 28 and 19 is 16. What is the value of x?

Definition 4.1: The mean of a set of data is the sum of all values divided by the number of values:

$$\text{mean} = \frac{\text{sum of all values}}{\text{number of values}}.$$

Example 8. Find the mean of 6, 14, 10, 14, 14, 12, 8, 2.

Solution: The sum of the values is: 6 + 14 +10 +14 + 14 + 12 + 8 + 2 = 80.

There are 8 values, so divide 80 by 8.

$$\text{Thus, mean} = \frac{\text{sum of all values}}{\text{number of values}}$$

$$= \frac{80}{8} = 10$$

The mean is 10

Example 9. The mean of three numbers is 10, and the mean of four other numbers is 16. What is the mean of all seven numbers?

Solution: For the first set of data

We get, sum of values = mean × number of values

Sum of values = 10 × 3

Sum of values = 30

For the second set of data

Similarly sum of values = mean × number of values

= 16 × 4

= 64

Thus total sum = 30 + 64 = 94

And total number of data = 3 + 4 = 7.

Therefore, mean of all seven numbers $= \frac{\text{Total sum}}{7}$

$= \frac{94}{7}$

$= 13.4$

Therefore, the mean of all seven number is 13.4.

Example 10. The mean of four numbers is 9. Three of the numbers are 8, 16 and 6. Find the value of the other number.

Solution: let x be the missing number.

Thus mean $= \frac{\text{Sum of all values}}{\text{number of values}}$

$9 = \frac{8+16+6+x}{4}$

$30 + x = 36$

$x = 6$

The missing number is 6.

Exercise 4E

1. Calculate the mean for each set of data.

a.	12	18	9	14	8	7		
b.	23	15	37	26	16	21	33	23
c.	15	25	22	34	19	20		
d.	25	12	31	26	31	19	30	
e.	60,	75,	95,	80,	200			

2. The heights of a group of students, in centimeters, are 158, 162, 172, 157, 161.
   a. Calculate the mean height.
   b. Another student joins the group. His height is 169 cm. calculate the new mean height.
3. The mean of four numbers is 94, and the mean of another nine different numbers is 17. What is the mean of all thirteen numbers?
4. Find the value of x so that the mean of the given data: 14, 6, 2x, 8, 10, 4 is 8.
5. What number should be included in the data 2, 8, 7, 4 and 9 so that the mean is 6?

Challenge Problems

6. If the mean of A and B is 20, the mean of B and C is 24 and the mean of A, B and C is 18. What is the mean of A and C?
7. If $2x^4 + 2y^4 + 2z^4 = 144$, what is the mean of x^4, y^4 and z^4?
8. A student has an average score of 90 on four tests. If the student scored 88, 96 and 92 on the first three tests. What was the students score on the fourth tests?
9. The mean of 5 numbers is 11. The numbers are in the ratio 1:2:3:4:5. Find the smallest number.
10. The mean length of 6 rods is 44.2cm. The mean length of 5 of them is 46 cm. How long is the sixth rod?

4.3.2. The Mode

Activity 4.3

Discuss with your teacher orally in the class

1. Find the mode of these sets of data.

a.	4	8	10	12	16	30	10	9		
b.	24	23	22	25	24	0	24	25	26	25
c.	26	29	60	70	80	60	70	80	100	
d.	200	600	700	800	900	1000				

Definition 4.2: The mode of a set of data is the value which occurs most frequently.

Note:

i. A data that has a unique mode is called unimodal.
ii. A set of data which has two modes is called bimodal.
iii. A set of data has three modes is called Trimodal.
iv. Each value occurs only once, so there is no mode at all.
v. The mode can usually be determined by observation.

Example 11 Find the mode of these sets of data.

a.	8	16	18	20	24	32	60	20		
b.	48	64	44	50	48	0	48	50	52	50
c.	300	400	150	900	250	350				

Solution:

a. The mode is 20, since it occurs more frequently than any other values of the data. Note that 20 occurs two times, which is more than any of the other numbers of the given data.

b. The number 48 occurs three times and the number 50 occurs three times. Hence, there are two modes 48 and 50.

c. Each value occurs only once, so there is no mode for the given data.

Exercise 4F

1. Calculate the mode of the following sets of numbers.

a.	200	406	406	609	708				
b.	326	580	580	799	799	900	900		
c.	1100	966	688	499	366	1278	1000	699	566
d.	1106	1207	1138	1166	1188	1196	1278	1179	
	1186	1186	1138						

4.3.3. The Median

Group work 4.3

Discuss with your group member.

1. Find the median of these numbers.

a.	2	3	4	8	12	13	14	18	19			
b.	3	8	8	9	10	12	14	18	21	23	25	30

Definition 4:3. The **median** is the middle value when the data is arranged in order of size.

Note: The median for a set of data with a total of n values is found by arranging the data in order from the smallest to the largest or from the largest to the smallest.

Example 12 Find the median of the population function whose values are:

a. 6 12 2 0 4 10 4 6

b. 18 4 2 18 14 8 4 6 12

Solution:

a. Arranged in increasing order: 0 2 4 4 6 6 10 12

8 data items ⇒ even items.

The two middle values are the 4^{th} and 5^{th} elements of the list which are 4 and 6. The median is half the sum of 4 and 6.

So the median of the even items = $\frac{4+6}{2}$

= 5

Therefore, the median is 5

b. Arranged in increasing order: 2 4 4 6 8 12 14 18 18

9 data items ⇒ odd items

And the middle value is the 5^{th} element of the list which is 8.

So the median of the odd items is 8

Hence the median is 8.

Example 13. Given a population function values: 10, -6, 4, -2, 7. What number must be included in the data so that the median will be 5.2?

Solution

population function values: 10, -6, 4, -2, 7

Median = 5.2

Numerical order: -6, -2, 4, x, 7, 10 since 4 < 5.2 < 7

$$\text{Median} = \frac{4+x}{2} = 5.2$$

$$\Rightarrow 4 + x = 10.4$$

$$\Rightarrow x = 6.4$$

Therefore, the included number is 6.4 .

Exercise 4G

1. Use the information given to find the value of x in each of the following sets of numbers.
   a. 2, x, 5, 7, 1, 3: the median is $\frac{7}{2}$.
   b. 4, 7, 2, x, 2, 9, 6: the median is 5
2. Find the median of these numbers.
   a. 38, 35, 35, 35, 30, 29, 28, 28, 11, 5
   b. 1, 3, 17, 18, 19, 20, 21, 21, 24

4.3.4. The Range

Activity 4.4

Discuss with your Friends.

1. Find the range of these sets of data.

a. 4	8	9	10	11	15	16	25	28	0
b. 10800	15000	15500	18300	21300					
c. -900	-200	-700	0	-1000					

2. The range of a set of data is 32. If the biggest data value is 52, find the smallest value.

Definition 4.4. The range of a set of data is the difference between the highest value and the lowest value:
the range = highest value – lowest value.

Example 14. Find the range of these sets of data.

a. -100, 600, 900, 500, 700   b. -600, 0, -2000, -1000, -8000

Solution:

a. Range = highest value – lowest value
= 900 –(-100)
= 900 + 100
= 1,000

b. Range = highest value – lowest value
= 0 – (-8000)
= 0 + 8000
= 8000

Example 15. The range for an English test was 70. What was the highest point, if the lowest had been 20.

Solution:

Range = highest value – lowest value

70 = highest value – 20

Highest value = 90

Exercise 4H

1. Find the range of the following mathematics examination scores.
   80 65 84 73 90 96
2. In a class of 30 students the highest score in physics test was 98 and the lowest was 35. What was the range?
3. Find the range of these sets of data: -2, -9, -1, -2000, -6000.
4. The range for the eight numbers shown is 40.
   Find the two possible values of the missing number.

13	5	27
?	19	?
42	11	33

Summary For Unit 4

1. A database is an organized collection of information. It can be stored on paper or computer.
2. Line graph is most commonly used to picture how two sets of data are related to each other.
3. A pie charts are also called a circle graphs.
4. Pie chart is a very common and accurate way of representing data specially useful for showing the relations of one item with another and one item to the whole item.
5. A good graph should contain the following points:
   a) Have a title.
   b) Be well proportioned.
   c) Have scales clearly marked and labeled.
   d) Show source of the facts that it represents.
6. The mean of a set of data is the sum of all the values divided by the number of values:

   $$\text{Mean} = \frac{\text{sum of values}}{\text{number of values}}$$
7. The mode of a set of data is the value which occurs most frequently.
8. The median is the middle value when the data is arranged in order of size.
9. The range of a set of data is the difference between the highest value and the lowest value:

   Range = highest value – lowest value.

Miscellaneous Exercise 4

I. Fill each of the following blank space with the appropriate Terminology.

1. The difference between the maximum and minimum value is called _______________.
2. An arrangement of data in an ascending or descending order is called _______________ .
3. _________, ___________ and ________ are called measure of location while __________ is measure of variation.
4. A set of data which has two modes is called ____________ .
5. _________ graphs can be used to show continuous data.

II. Work out questions

6. A survey was conducted by asking 120 students in a town how they traveled to school. The following pie chart shows the result of the survey.

   What are the number of students that travel to school by bus?

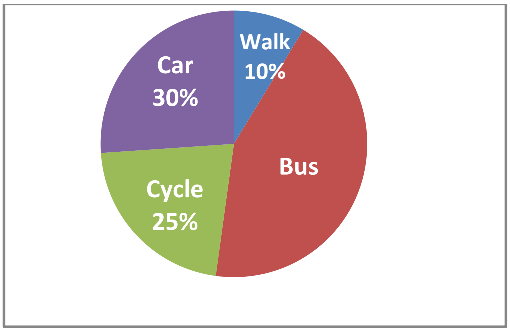


Figure 4.19 Pie chart

7. 3000 students appeared for an examination from five different centres C_1, C_2, C_3, C_4 and C_5 of a city. From the given pie chart, find the number of students appearing for the examination from each centre.

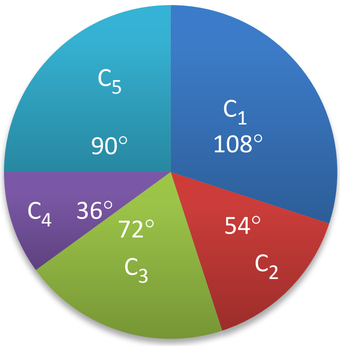


Figure 4.20 Pie chart

8. The following table shows the distribution of 90 apprentice workers in a factory according to trade (See Table 4.14).

Trade	Fitting	Turning	Welding	Molding	Spray painting
Number	25	30	8	15	12

Show the above data by a pie chart

9. A teacher listed 28, 30, 32 and 36 as ages of students in his class with frequencies 8, 10, 5 and 7 respectively.
   a. How many students were in the class?
   b. What was the average age of the class?
   c. What was the range for the students?
   d. What was the modal age?
10. Given the data 4, y, 9, 5, 2, 7. Find y if
    a. the mean is 5 b. the median is 6 c. the mode is 4
11. The mean of six numbers is 12. Five of the numbers are 11, 7, 21, 14 and 9. Calculate the sixth number.
12. Use the information given to find the value of n in each of the following sets of numbers.
    a. 5, 7, 4, 1, n, 5: the mean is 6
    b. 3, 1, 4, 5, 4, n : the mode is 4
    c. 1, 7, 2, 1, n, 4, 3: the modes are 1 and 2
    d. 2.6, 3.5, n, 6.2: the mean is 4
13. Find five numbers so that the mean, median, mode and range are all 4.
14. The mean of 3,7,8,10 and x is 6. Find x.
15. Write down five numbers so that: the mean is 6, the median is 5 and the mode is 4.
16. Find the mean, median and mode of these sets of data:
    a. 14,12,24,36,23
    b. 114,112,124,136,123
    c. 2,3,4,5,30
    d. x, 2x,3x,4x,5x
17. a) The mean of 5 numbers is 8. Four of the numbers are 7,9,11 and 5. Find the fifth number.
    b) The mean of 4,8,9,x and 2x is 6. Calculate the value of x.

UNIT 5

GEOMETRIC FIGURES AND MEASUREMENT

Unit outcomes:

After completing this unit, you should be able to:

- identify, construct and describe properties of quadrilaterals such as trapezium and parallelogram.
- identify the difference between convex and concave polygons.
- find the sum of the measures of the interior angles of a convex polygon.
- calculate perimeters and areas of triangles and trapeziums.

Introduction

In this unit you will extend your knowelede of gemotric figures. You will exercise how to construct quadrilaterals and describe their properties using your construction. You will also learn more about triangles. Moreover you will be able to calculate the areas and perimeter of Plane figures including solid figures like surface areas and volumes of prisms and circular cylinders.

5.1 Quadrilaterals, Polygons and Circles

The purpose of this section is, to enable you to construct and to let you know the basic facts about **quadrilaterals**, **polygons** and **circles**.

5.1.1 Quadrilateral

Growp Work 5.1

Discuss the following key terms with friends /Groups/.

1. List the three basic terms in plane geometry.
2. Define the following key terms and explain in your own word:
   a. line segment
   b. ray
   c. angles
   d. adjacent angles
   e. vertically opposite angles
   f. angle bisectors
   g. complementary angles
   h. supplementary angles
3. Look at Figure 5.1 below
   a. Name all its vertices.
   b. Name all its interiaor angles.
   c. Name all its sides.
   d. Name all pairs of opposite sides.
   e. Shows the number of all possible diagonals that can be drawn from all its vertices.

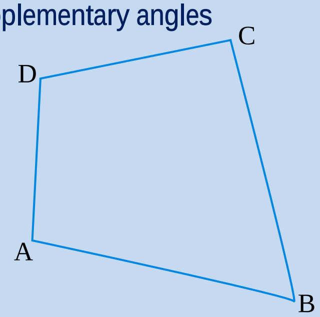


Figure 5.1. Quadrilateral

? Have you any idea on how to name Figure 5.1 above?

Note: Therefore, the Figure given above (Figure 5.1) is a quadrilateral and this quadrilateral is denoted by using and naming all the letters representing its vertices either in clockwise or counterclockwise directions. Thus, we can name this quadrilateral as quadrilateral ABCD or BCDA or CDAB or DABC or DCBA or remember that it can not be named as ACBD or BDAC.

Definition 5.1: A quadrilateral is a four-sided geometric figure bounded by line segments.

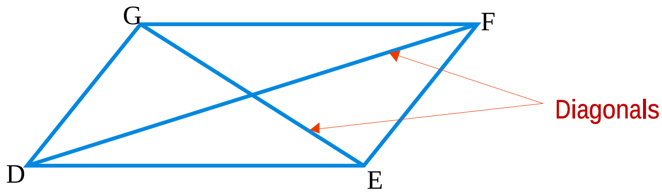


Figure 5.2. Quadrilateral DEFG

Note: i) The line segments $\overline{DE}, \overline{EF}, \overline{FG},$ and $\overline{GD}$ are called sides of the quadrilateral DEFG.

ii) The points at which the sides are connected are vertices of the quadrilateral. In Figure 5.2 the points D, E, F and G are vertices of the quadrilateral.

iii) Adjacent sides of a quadrilateral are sides that have a common end point. In Figure 5.2 the sides $\overline{DE}$ and $\overline{EF}$ are adjacent sides since they have met at vertex E.

iv) Opposite sides are sides that have no common point, and $\overline{DG}$ and $\overline{EF}$, $\overline{DE}$ and $\overline{GF}$ are opposite sides; because they have no common vertex.

v) A diagonal is a line segment that connects two opposite vertices. In Figure 5.2 $\overline{DF}$ and $\overline{EG}$ are diagonals of the quadrilateral.

vi) The interior angles of a quadrilateral are the angles formed by adjacent sides of the quadrilateral and lying with in the quadrilateral.

In Figure 5.2, the quadrilateral contains four interior angles ∠D, ∠E, ∠F and ∠G or $\hat{D}$, $\hat{E}$, $\hat{F}$ and $\hat{G}$.

5.1.1.2 Construction and Properties of Trapezium

Activity 5.1

Discuss with your teachers before starting the lesson.

1. This shape is a quadrilateral. Can you name the shape of this quadrilateral?

2. Is there any thing that you can say about the pairs of its opposite sides.

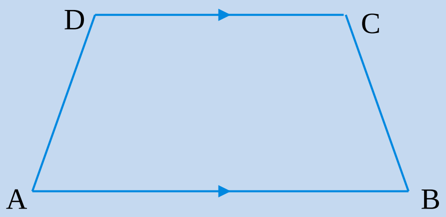


Figure 5.3.

3. Construct a quadrilateral ABCD with AB||CD and AB = 6cm, BC = 3cm $m(\angle A) = 50^o$ and $m(\angle B) = 80^o$.

In numbers 4-6 construct a trapezium ABCD in which AB is parallel to DC.

4. If AB = 8cm, BC = 4cm, CD = 3cm and DA = 3.5cm, then find measure $\angle$ A.

5. If AB = 5cm, BC = 6cm, CD = 2cm and DA = 4cm, then find neasure $\angle$A.

6. If AB = 6.5cm, CD = 3cm, AC = 7cm and BD = 5cm, then describe shortly your method.

7. Construct the parallelogram ABCD, given that AB = 7cm, AC = 10cm and BD= 8cm. What is the measure of $\overline{BC}$.

To perform geometric constructions; you need **a straight edge** and **compass**. Using these basic tools; you can construct a geometric figure with sufficient accuracy.

- ✓ **Use of a straight edge:** A straight edge marked or unmarked, ruler is used to construct (draw) a line or a line segment through two given points.
- ✓ **Use of compasses:** is used to construct (draw) circles or arcs.

Note: To draw a figure you may use any convenient instrument such as ruler, protractor etc.

? Is there a difference in meaning between the word "drawing" and construction?

Definition 5.2: A trapezium is a special type of a quadrilateral in which exactly one pair of opposite sides are parallel.

- The parallel sides are called the bases of the trapezium.
- The distance between the bases is known as the height (or altitude) of the trapezium.

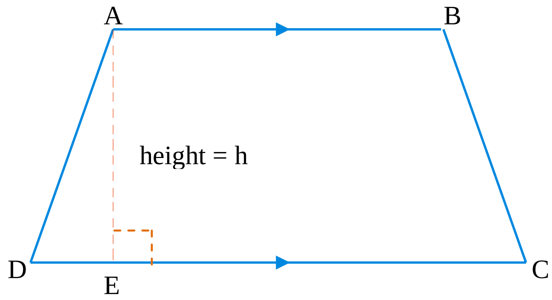


Figure 5.4 Trapezium

- In Figure 5.4 quadrilateral ABCD is a trapezium with bases $\overline{AB}$ and $\overline{DC}$. $\overline{AB} \parallel \overline{DC}$ and the distance between $\overline{AB}$ and $\overline{DC}$ is the height of trapezium ABCD.
- In Figure 5.4 $\overline{AD}$ and $\overline{BC}$ are the non – parallel sides of the trapezium called the **legs of the trapezium.**

Construction I

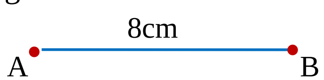
Construct a trapezium ABCD using ruler, protractor, pair of compasses and the given information below.

Given: AB||CD, AB = 8cm, BC = 5cm, $m(\angle A) = 60°$ and $m(\angle B) = 85°$.

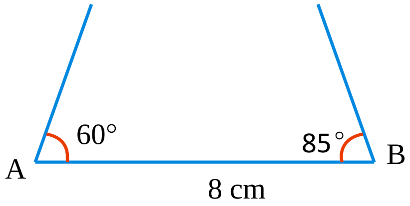
Required: To construct trapezium ABCD.

Solution:

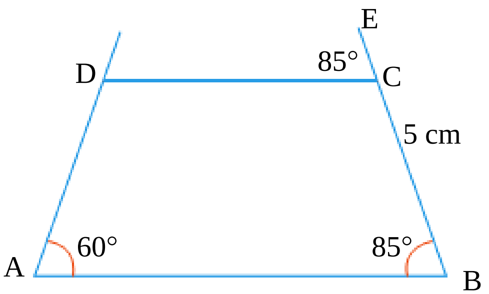
Step i: Draw a line segment AB = 8cm.



Step ii: Construct m (∠A) and m (∠B) with the given measures.



Step iii: Mark point C on the side of ∠B such that BC = 5cm.



Step iv: Draw a line through C and parallel to $\overline{AB}$ so that it intersects the side of ∠A at point D.

Therefore, ABCD is the required trapezium.

5.1.1.3 Construction and Properties of Parallelogram

Definition 5.3: A paralleloagram is a quadrilateral in which each sides is parallel to the side opposite to it.

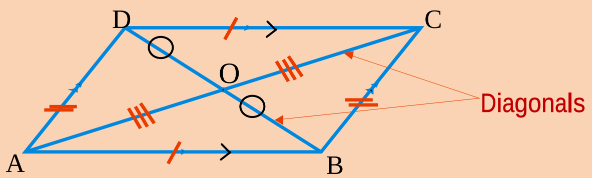


Figure 5.5 parallelogram

In Figure 5.5 $\overline{AB}||\overline{DC}$ and $\overline{AD}||\overline{BC}$, thus, ABCD is a parallelogram.

Construction II

Construct parallelogram ABCD using ruler, protractor, pair of compasses and information given below.

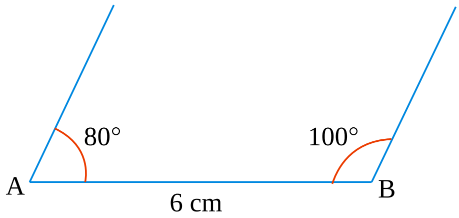
Given: $\overline{AB} \| \overline{CD}$, AB=6cm, BC=4cm and m(∠A)= 80° .

Required: To Construct parallelogram ABCD.

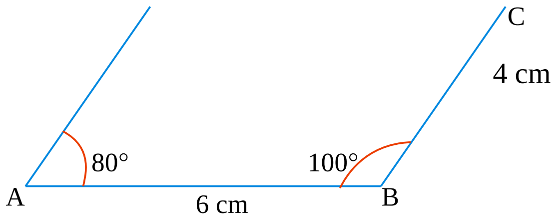
Solution:

Step i: Draw a line segment AB = 6cm.

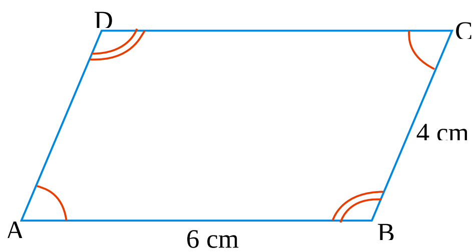
Step ii: Construct ∠A and ∠B so that m(∠A) = 80° and m(∠B)= 100°.



Step iii: Mark point C on side of ∠B such that BC = 4cm.



Step iv: Draw a line through C and parallel to $\overline{AB}$ so that it meets the side of ∠A at point D.



Therefore ,ABCD is the required parallelogram.

Properties of parallelogram

i. Opposite sides of a parallelogram are congruent. In Figure 5.5 ABCD is a parallelogram, then AB = CD and AD = BC.

ii. Opposite sides of a parallelogram are parallel. In Figure 5.5 ABCD is a parallelogram then $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$.

iii. Opposite angles of a parallelogram are congruent. In Figure 5.5 ABCD is a parallelogram then $m(\angle A) = m(\angle C)$ and $m(\angle B) = m(\angle D)$.

iv. Consecutive angles of a parallelogram are supplementary. In Figure 5.5 ABCD is a parallelogram then $m(\angle A) + m(\angle B) = 180°$, $m(\angle B) + m(\angle C) = 180^o$ etc.

v. The diagonals of a parallelogram bisect each other. In Figure 5.5 ABCD is a parallelogram and the diagonals $\overline{AC}$ *and* $\overline{BD}$ intersect at O then AO = CO and BO = DO.

Note: Bisect means "divides exactly into two equal parts".

Example 1. Find the values of x and y in parallelogram ABCD. Then find AE, EC, BE, and ED.

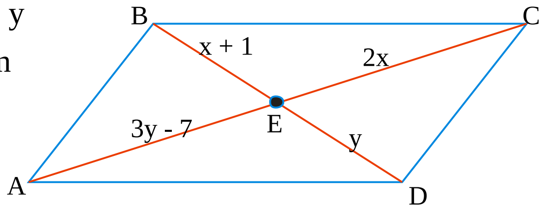


Figure 5.6 parallelogram

Solution:

AE = CE…… The diagonals of a parallelogram bisect each other.

$3y - 7 = 2x$ ….. Equation 1

DE = BE…. The diagonals of a parallelogram bisect each other.

$x + 1 = y$ ….. Equation 2

$3(x + 1) - 7 = 2x$ …… Substitute equation 2 in equation1

$3x + 3 - 7 = 2x$ ……. Remove brackets

$3x - 4 = 2x$ ………… Simplifying

$3x - 4 + 4 = 2x + 4$ ….. Adding 4 from both sides

$3x = 2x + 4$ …… Simplifying

$3x - 2x = 2x - 2x + 4$ ….. Subtracting 2x from both sides

$x = 4$ units.

when $x = 4$

thus $y = x + 1$

$y = 4 + 1$

$y = 5$ units.

Therefore, $AE = 3y - 7$

$= 3(5) - 7$

$= 15 - 7$

$= 8$ units.

Therefore, $EC = 2x$

$= 2(4)$

$= 8$ units.

Therefore, $BE = x + 1$

$= 4 + 1$

$= 5$ units.

Therefore, $DE = y$

$= 5$

Hence $AE = EC$ and $BE = DE = 5$ units.

Example 2. In Figure 5.7 to the right ABCD is a parallelogram. Find the measure of $\angle A$, $\angle B$ and $\angle C$.

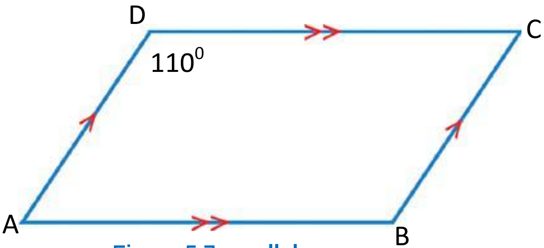


Figure 5.7 parallelogram

Solution:

$m(\angle B) = m(\angle D) = 110^o$ since measures of the opposite angles of a parallelogram are equals.

$m(\angle A) + m(\angle D) = 180^o$ ….. Consecutive angles of a parallelogram are suppalementary.

$m(\angle A) + 110^o = 180^o$ …. Substitution

$m(\angle A) + 110^o - 110^o = 180^o - 110^o$ …. Subtracting 110^o from both sides

$m(\angle A) = 70^o$ …. Simplifying

$m(\angle A) = m(\angle c)$ ….. Opposite angles of a parallelogram are congruent (have equal measure).

$m(\angle c) = 70^o$

Exercise 5A

1. In Figure 5.8 on the right, shows a parallelogram ABCD is given.
   If the diagonals $\overline{AC}$ and $\overline{BD}$ intersect at O and AO = 4cm, find the length of $\overline{AC}$.

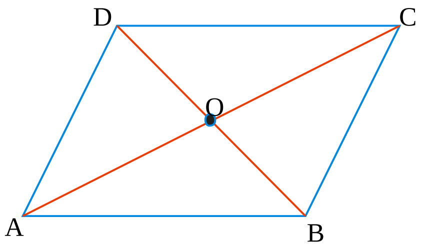


Figure 5.8 parallelogram

2. In Figure 5.9 below ABCD is a parallelogram with m(∠ABC) = 43°. A line through A meets $\overline{CD}$ at E and m(∠AED) = 68°.
   Find
   a. m(∠ADE)
   b. m(∠ DAE)
   c. m(∠BCD)

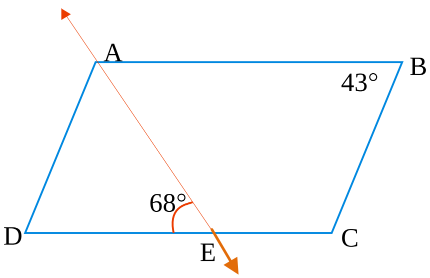


Figure 5.9 parallelogram

3. In Figure 5.10 find the unknown marked agnles.

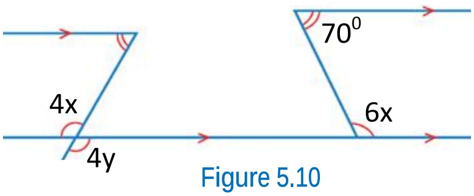


Figure 5.10

In exercises, 4 and 5, find the unknown or marked angles.

4.

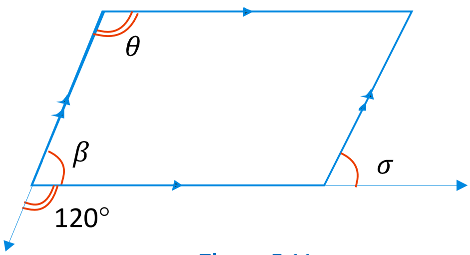


Figure 5.11

5.

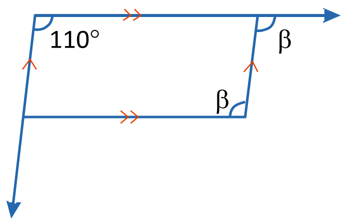


Figure 5.12

Challenge Problem

In exercises 6 and7, find the unknown or marked angles.

6.

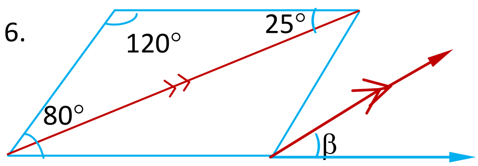


Figure 5.13

7.

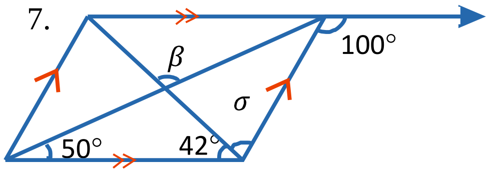


Figure 5.14

5.1.1.4 Construction and Properties of Special Parallelogram

A. Rectangle

Activity 5.2

Discuss with your calssmate

1. Construct a rectangle PQRS with PQ = 4cm, QR = 3cm and m(∠P) = 90°.
2. Name the following
   a. The green side of a rectngle
   b. The blue side of a rectangle
   c. The red diagonal of a rectangle

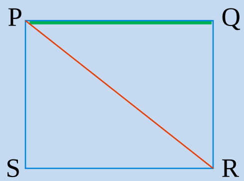


Figure 5.15 Rectangle

3. a. Draw accurately rectangle ABCD where AB = 4cm, BC = 3cm.
   b. Join the diagonal $\overline{\mathbf{BD}}$ and give its length.
4. Construct the rectangle ABCD, given thath AB = 4cm and AC = 6cm.
5. Construct the square ABCD, given that AC = 5cm. What is the measure of AB.
6. Construct the rhombus ABCD given thath BD = 7cm , ∠B = 40°. What is the measure of AC .

Construction III

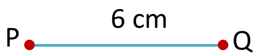
Construct a rectangle PQRS by using ruler, protractor, pair of compasses and the given information below.

Given: $\overline{PQ}\,||\,\overline{RS}$,PQ =6cm, QR=7cm and m(∠P)=90°.

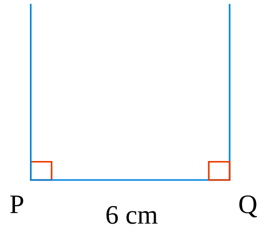
Required: To construct rectangle PQRS.

Solution:

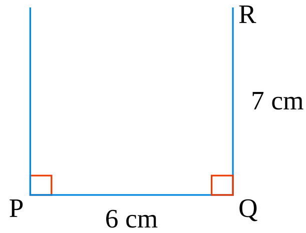
Step i: Construct a line segment $\overline{PQ}$ with length 6cm.



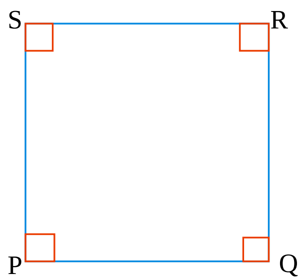
Step ii: Construct $m(\angle P) = 90^{o}$ and $m(\angle Q) = 90^{o}$.



Step iii: Mark point R such that QR = 7cm.



Step iv: Draw a line through R and parallel to $\overline{PQ}$ so that it interects with a line through P and parallel to $\overline{QR}$. Let S be the intersection point.



Therefore PQRS is the required rectangle.

Definition 5.4: A rectangle is a parallelogram with all its angles are right angles.

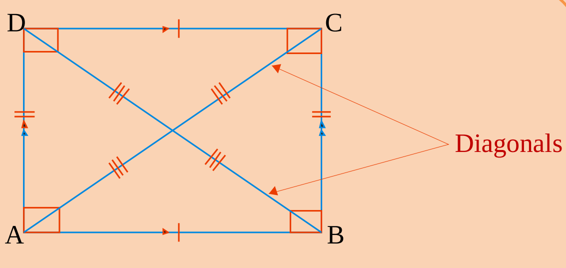


Figure 5.16 Rectangle

Properties of a rectangle

i. Arectangle has all properties of a parallelogram.

ii. All angles of a rectangle are right angles.

iii. The diagonals of a rectangle are equal in length and bisect one another. That is, if ABCD is a rectangle then AC = BD.

iv. The consecutive angles of a rectangle are equal. That is, if ABCD is a rectangle, then $m(\angle A) = m(\angle B) = m(\angle C) = m(\angle D) = 90^o$.

Note: A quadrilateral with congruent diagonals is not necessarily a rectangle.

Exercise 5B

1. In Figure 5.17 to the right ABCD is a rectangle. If $m(\angle BDC) = 54°$, then find $m(\angle ABD)$ and $m(\angle CBD)$.

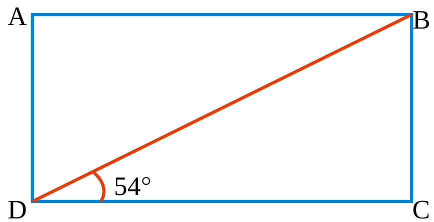


Figure 5.17 Rectangle

2. In rectangle ABCD the length of diagonal $\overline{AC}$ is given by (20x + 12) cm and the length of diagonal $\overline{BD}$ is given by (14x + 24)cm. Find AC and BD.

3. In Figure 5.18 to the right EFGH is a rectangle. If $m(\angle HFG) = 37^o$. what is the value of β.

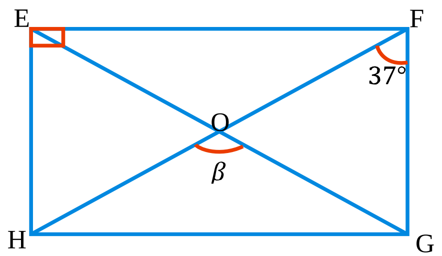


Figure 5.18 Rectangle

4. In Figure 5.19 to the right PQRS is a rectangle. If PS = 5 cm and PR = 13cm, find SR and QS.

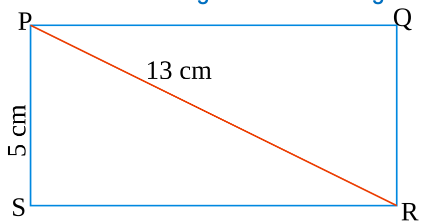


Figure 5.19 Rectangle

5. Construct the rectangle EFGH with EF = 6cm FG = 3cm. Describe its construction.

B. Rhombus

Activity 5.3

First discuss for each step with your friends and ask your teacher.

1. Construct a rhombus ABCD with AB = 4cm and m(∠A) = 70°.
2. Give your own example similar to Activity 5.4 here on number 1 above and show for each step and arrive on the final conclusion.

Definition 5.5: A rhombus is a parallelogram in which two adjacent sides are congruent.

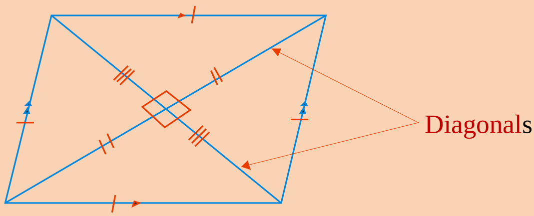


Figure 5.20 Rhombus

Properties of rhombus

i. All sides of a rhombus are equal (congruent).
ii. Opposite sides of a rhombus are parallel.
iii. Opposite angles of a rhombus are equal (congruent).
iv. The diagonal of a rhombus bisect each other at right angles.
v. The diagonal of a rhombus bisects the angles at the vertices.

Example 3. In Figure 5.21 to the right ABCD is a rhombus if AB = 12cm, then find DC.

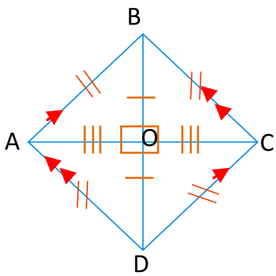


Figure 5.21

Solution:

By propertiy (i) AB = DC = 12cm.

C. Squares

Activity 5.4

Discuss with your teacher in the class based on the above discussion.

1. Construct a square ABCD with AB = 4cm, m(∠A) = 90°.
2. Based on activity number 1 ask each student to give their final conclusion.

Definition 5.6: A square is a rectangle in which its two adjacent sides are congruent.

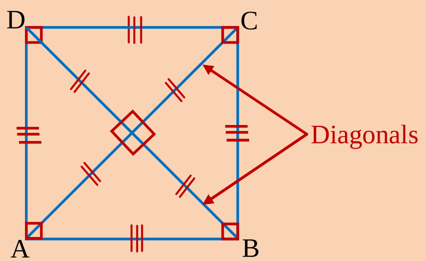


Figure 5.22 Square

Properties of Square

i. All the sides of a square are equal (Congruent).

ii. All the angles of a square are right angles.

That is m(∠A)= m(∠B) = m(∠C) = m(∠D) = 90°.

iii. Opposite sides of a square are parallel. That is $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$.

iv. The diagonals of a square are equal (Congruent) and perpendicular bisectors of each other.

v. The diagonals of a square bisect the angles at the vertices.

Example 4. In Figure 5.23 to the right ABCD is a square. $\overline{CO}$ intersects $\overline{DB}$ at E. If the measure of ∠DEC = 70°, then find the measure of ∠AOC.

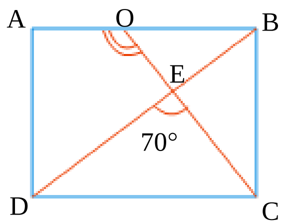


Figure 5.23 square

Solution:

Since each angle of a square is bisected by a diagonal

$m(\angle ABD) = \frac{1}{2}(90^o) = 45^o$

$m(\angle BEO) = m(\angle DEC) = 70^o$ ………… Vertical opposite angle.

Thus $m(\angle BOE) + m(\angle OEB) + m(\angle EBO) = 180^o$ why?

$m(\angle BOE) + 70^o + 45^o = 180^o$ ….. Substitution

$m(\angle BOE) = 180^o - 115^o$

$m(\angle BOE) = 65^o$.

Now $m(\angle AOC) + m(\angle BOC) = 180^o$ …….. Supplementary angles.

$m(\angle AOC) + 65^o = 180^o$ ……. Substitution

$m(\angle AOC) = 115^o$

Exercise 5C

1. Find the length of the side of a rhombus whose diagonals are of length 6cm and 8cm

2. In Figure 5.24 to the right ABCD is a rhombus. Show that $\overline{AC}$ is the bisector of $\angle BAD$.

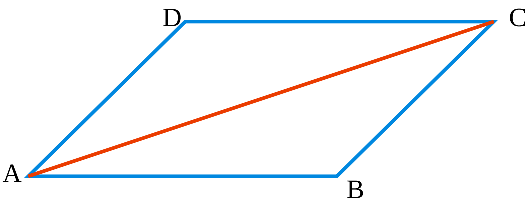


Figure 5.24 Rhombus

3. In Figure 5.25 to the right shows ABCD which is a rhombus; with $m(\angle BAD) = 140^o$. Find $m(\angle ABD)$ and $m(\angle ADC)$.

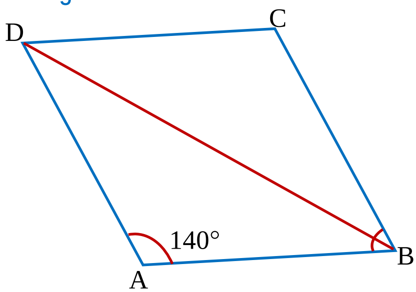


Figure 5.25 Rhombus

4. In Figure 5.26 to the right, ABCD is asquare. Find the measure of $\angle ABD$.

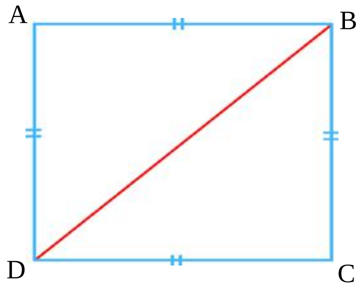


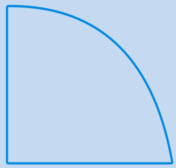
Figure 5.26 square

5.1.1. Polygons

In this subunit you will see the different types of polygons, simple, convex and concave polygons. But most of our discussion will be on convex and concave polygons. Polygons are classified according to the number of sides they have.

Activity 5.5

1. This shape is not a polygon.
   Explain why.



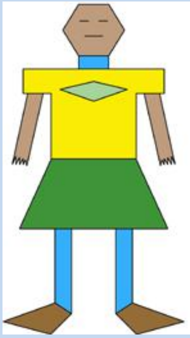
2. Identify the given shapes is a convex or a concave polygon.

(a) (b) (c) (d)

Figure 5.27

3. The following pictures are made from polygons. Copy the tables below and fill the blank space correctly.

(a) Picture A



(b) Picture B

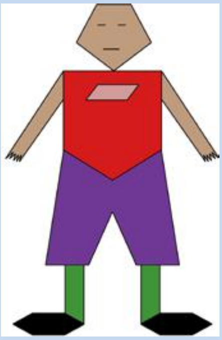


Figure 5.28

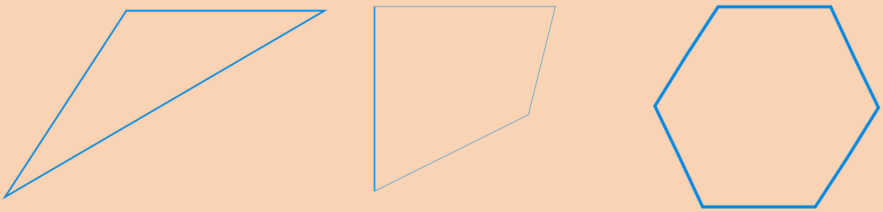
For picture A			For picture B		
Description	Number of sides	Name of polygon	Description	Number of sides	Name of polygon
1. Neck 2. Head 3. T-shirt 4. Name bage 5. Skirt 6. Shoes	______ ______ ______ ______ ______ ______	______ ______ ______ ______ ______ ______	1. Head 2. T-shirt 3. Name badge 4. Trousers 5. Legs 6. Shoes	______ ______ ______ ______ ______ ______	______ ______ ______ ______ ______ ______

Definition 5.7: Apolygoan is a simple closed plane figure formed by three or more line segments joined end to end.

The line segments forming the polygons are called **sides** and the common end point of any two sides is called **Vertex (plural vertices)** of the polygon. The vertices of a polygon are the points where two sides meet.

A. Convex and concave polygons

Definition 5.8: A convex polygon is a simple polygon in which all of its interior angles measures less than 180^o each.



(a) (b) (c)

Figure 5.29 Examples of convex polygons

Definition 5.9: A concave polygon is a simple polygon which has at least one interior angle of measures greater than 180°.

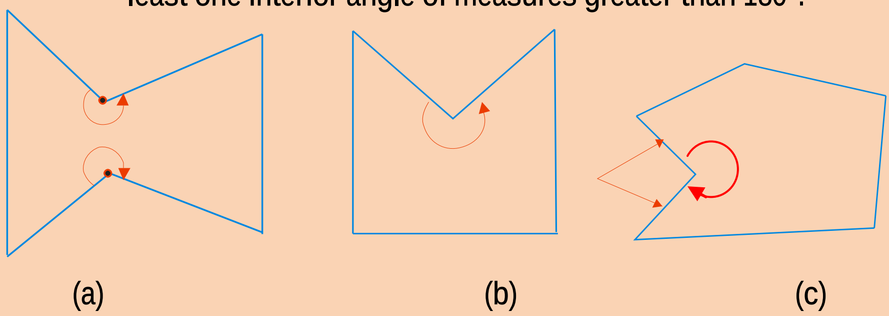


Figure 5.30 Examples of concave polygons

Definition 5.10: A diagonal of aconvex polygon is a line segment whose end points are non-consecutive vertices of the polygan.

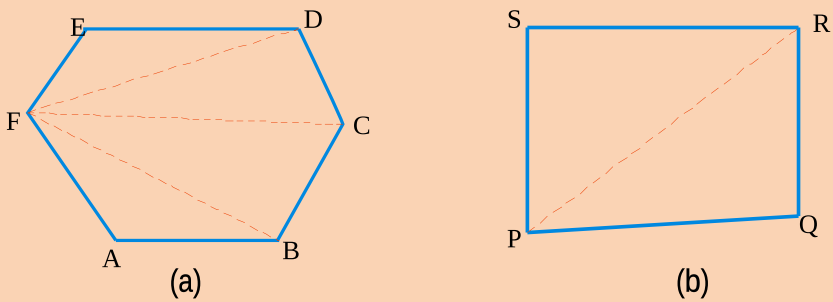


Figure 5.31 Shows the number of diagonals that can be drawn from one vertex.

Vertex of agiven polyon. In Figure 5.31 (a) $\overline{FD}$, $\overline{FC}$ and $\overline{FB}$ are the diagonal of the polygon from vertex F only and in Figure 5.31 (b), $\overline{PR}$ is the diagonal of the polygon from vertex P. A polygon is named by using the letters representing the vertices in clockwise or conunter clock wise direction.

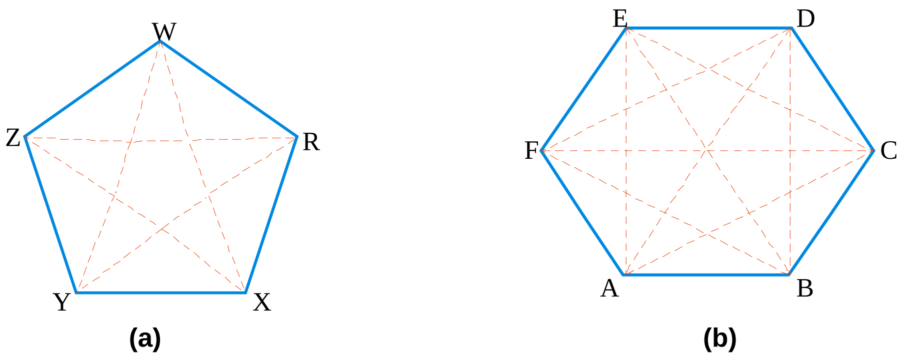


Figure 5.32 Shows the number of all possible diagonals that can be drawn from all vertices of the polygon XYZWR and ABCDEF.

Activity 5.6

1. Look at the polygons in Figure 5.32 above and list down all the possible diagonals in
   a. Polygon RXYZW.
   b. Polygon ABCDEF .
2. Draw an octagon and list down all the diagonals that can be drawn from all vertices. (Name the vertices A, B, C, D, E, F, G, H).

Table 5.1. Number of sides of a polygon and respective number of diagonals.

Number of sides	Number of diagonals drawn from one vertex	Number of all possible diagonals
3	0	0
4	1	2
5	2	5
6	3	9
7	4	14
8	5	20
9	6	27
10	7	35
n	n-3	$\frac{n(n-3)}{2}$

Example 5.: How many diagonals are there in a polygon of 40 sides?

Solution: Number of all possible diagonals = $\frac{n(n-3)}{2}$ ….. given formula

$$= \frac{40(40-3)}{2}$$

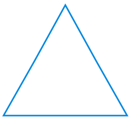
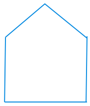
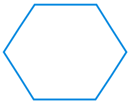
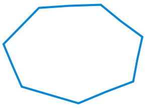
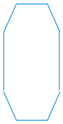
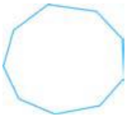
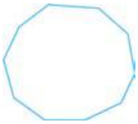
$$= \frac{40(37)}{2}$$

= 740 different diagonals.

A. Classification of polygons

Polygons are classified according to the number of sides they have. In Table 5.2 below is a list of some common types of polygons and the number of sides of each polygon.

Table 5.2 Types of polygons

Number of sides	Figure	Name of the polygon
3		**Triangle**
4		**Quadrilateral**
5		**Pentagon**
6		**Hexagon**
7		**Heptagon(septagon)**
8		**Octagon**
9		**Nonagon**
10		**Decagon**

Exercise 5D

Solve each of the following word problems.

1. How many possible diagonals are there in a polygon of 80 sides.
2. What is the number of sides of a Dodecagon?
3. What is the number of sides of an Icosagon?
4. Look at Figure 5.33 to answer the following questions.
   a. Name all vertices of the polygon.
   b. Name the opposite side of $\overline{AB}$.
   c. Name all diagonals that can be drawn from vertex B.
   d. How many interior angles does the polygon have?
   e. Name the polygon.

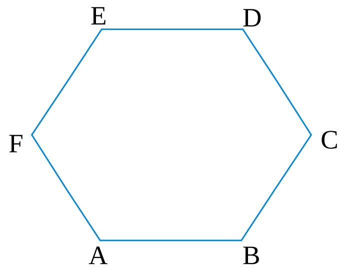


Figure 5.33 polygon

5.1.2. Circles

Group Work 5.2

Solve each of the following word problems.

1. Use compasses to draw your own circles.
2. Draw acircle of radius 3cm.

   In your circles draw and label.

   a. a diameter
   b. a radius
   c. a chord
   d. an arc
   e. a semicircle
   f. the circumference

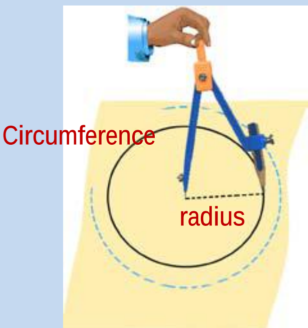


Figure 5.34

Definition 5.11: A circle is the set of all points in a plane that are equidistant from a fixed point called **the center of the circle**.

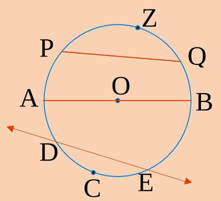


Figure 5.35 Circle

Note: i. a circle is usually named by its center. In Figure 5.35 the circle can be named as circle O.

ii. a chord of a circle is a line segment whose end points are on the circle. In Figure 5.35 the line segment $\overline{AB}$ **and** $\overline{PQ}$ are chords of the circle.

iii. a diameter of a circle is any chord that passes through the center, and denoted by ‘d’. It is the biggest chord of a circle. In Figure 5.35 the chord $\overline{AB}$ is a diameter of the circle.

iv. a radius of a circle is a line segment that has the center as one end point and a point on the circle as the other end point, and denoted by ‘r’. In Figure 5.35 the line sement $\overline{OA}$ **and** $\overline{OB}$ are radii of th circle, (radii is the prular from of radius).

v. circumfrence of a circle is the complete path around the circle.

From the above discussion AB = d = a diameter and O is the centre of the circle.

i.e AO = OB = r = radii.

Therefore, AB= AO + OB ….. The length of a segment equals the sum of the lengths of its parts that donot overlap.

d = r + r ……. Substitution

d = 2r ……. Collect like terms

Hence, the diameter ‘d’ of a circle is twice the radius r.

i.e. $d = 2r$ or $\frac{d}{2} = r$

Definition 5.12: An arc is a part of the circumference.

The part of the circle determined by the line through points D and E is called **an arc of the circle**. In Figure 5.35 we have arc DCE and arc DZE,

Notation: Arc DCE and arc DZE is denoted by $\overset{\frown}{DCE}$ and $\overset{\frown}{DZE}$ respectively where D and E are end points of these arcs.

Exercise 5E

Solve each of the following problems

1. In a circle of radius 3cm,
   a. draw a chord of 3cm.
   b. draw a chord of 6cm. what can you say about this chord?
   c. can you draw a chord of 7cm?
2. If in the following Figure 5.36 below O is the center of the circle, then
   a. ______ , ______, _____ and _____ are radii of the circle.
   b. ______ and ______ are diameter of the circle.
   c. _______ and ______ are chord of the circle.
   d. ______ and _______ a pair of parallel lines.

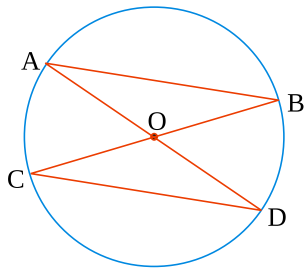


Figure 5.36

5.2. Theorems of Triangles

Group work 5.3

Solve each of the following word problems.

1. a. cut out a large triangle from scrape paper.
   b. Draw round the triangle in your book.
   c. Tear the three corners from your triangle made of the scrape paper.
   d. stick the torn angles inside its out line. (keep the cut out corners and stick them in a straight line or 180^o).

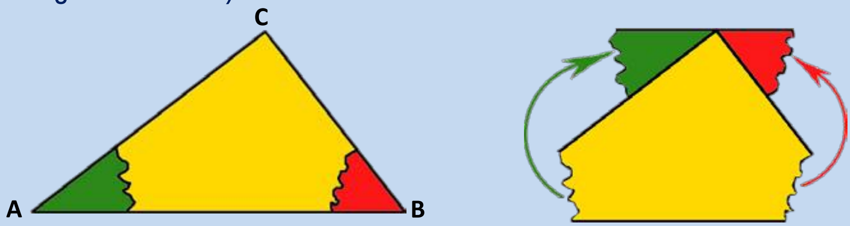


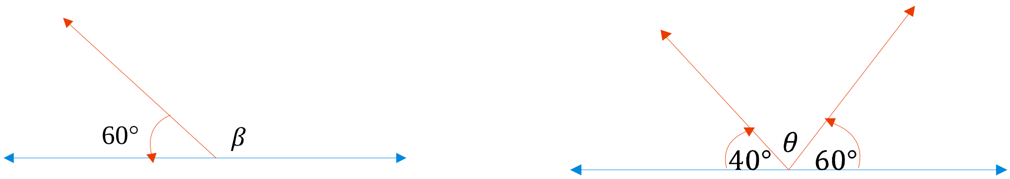
Figure 5.37

Finally what do you guess about the sum of the measures of interior angles of a triangle ABC.

Remember that: The following key terms are discussed in your grade six mathematics lessons.

Note: The angles on a straight line add up to 180^o.

Example 6. Calculate the marked angles in given Figures 5.38 below.



(a) Figure 5.38 (b)

Solution:

a. $60^o + \beta = 180^o$ …... Definition of straight angle

$= 60^o - 60^o + \beta = 180° - 60°$…. Subtracting 60 from both sides

$\beta = 120^o$ …….. Simplifying

b. $40^o + \theta + 60^o = 180^o$ …. Definition of straight angle

$\theta + 100^o = 180^o$

$\theta = 100^o - 100^o = 180° - 100°$ ….Subtracting 100 from both sides

$\theta = 80^o$ ….. Simplifying

Theorem 5.1: If two parallel lines are cut by a transeversal line, then alternate interior angles are equal.

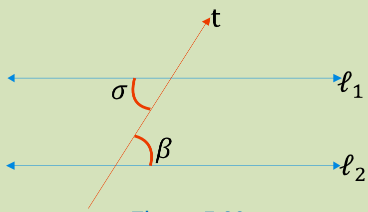


Figure 5.39

In Figure 5.39 σ and β are alternate interior angles.

Theorem 5.2: If two parallel lines are cut by a transversal line then, interier angles on the same sides of the transeversal line are supplementary.

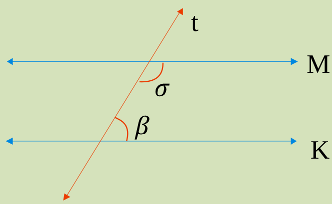


Figure 5.40

Theorem 5.3: If two parallel lines are cut by atransveral line, then corresponding angles are equal. In Figure 5.41 to the right if letters a, b, c, d, e, f, g and h represent the degree measures of the angles, then Theorem 5.3 states that:

f = c,

b = h,

e = d and

a = g.

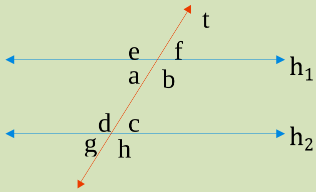


Figure 5.41

Theorem 5.4: (Angle – sum theorem)

The sum of the degree measures of the interior angles of atriangle is equal to 180º.

Proof: Let ABC be a triangle and $\propto$,β and γ be the measures of its interior angles.

We want to show that:

$\propto + \beta + \gamma = 180°$

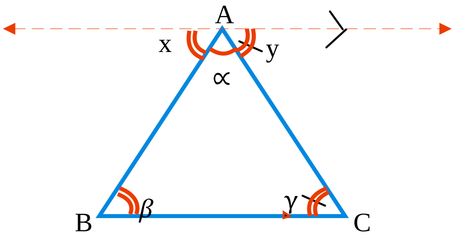


Figure 5.42 triangle

Statements	Reasons
1. Draw a line passing through A and parallel to $\overline{BC}$	1. Construction
2. $x + \alpha + y = 180°$	2. Definition of straight angle
3. $x = \beta$ and $y = \gamma$	3. Alternate interior angles
4. $\beta + \alpha + \gamma = 180°$	4. Substitution

Example 7. If the measures of the angles of a triangles are 2β, 3β and 4β, then give the measure of each angle.

Solution: Let the triangle be as shown in the Figure below,

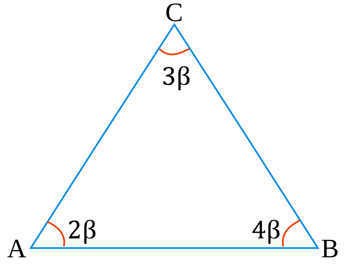
$m(\angle ACB) + m(\angle CBA) + m(\angle BAC) = 180^o$ why?

$3\beta+4\beta+2\beta= 180^o$ ……… Substitution.

$9\beta = 180^o$ ………. Collect like terms.

$\frac{9\beta}{9} = \frac{180^o}{9}$ ………. Dividing both sides by 9.

$\beta = 20^o$ ……….. Simplifying.



When $\beta = 20^o$

$m(\angle A) = 2\beta = 2(20^o) = 40^o$, $m(\angle C) = 3\beta = 3(20^o) = 60^o$ and

$m(\angle B) = 4\beta = 4(20^o) = 80^o$

Example 8. In Figure 5.43 below, if u°, v° and x° are degree measures of the angles marked, then what is the value of $m(\angle u) + m(\angle v)$?

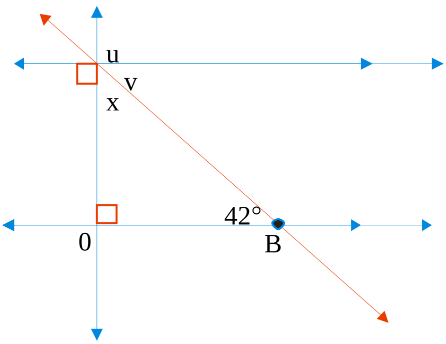


Figure 5.43

Solution:

$m(\angle O) + m(\angle x) + m(\angle B) = 180^o$ ….. Angle sum theorem.

$90^o + x + 42^o = 180^o$ ……Substitution.

$m(\angle x) + 132^o = 180^o$

$m(\angle x) = 48^o$

so $u = 90^o$ and $v = 42^o$

Therefore, $u + v = 90^o + 42^o$

$= 132^o$

Theorem 5.5: The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote (non adjacent) interior angles.

Proof: let ABC be a triangle with $\overline{AC}$ extended to form an exterior angle. Let $\propto$, β and γ be degree measures of the interior angles of triangle ABC and ω be the degree measure of the exterior angle.

We want to show that: $\propto + \beta = \omega$

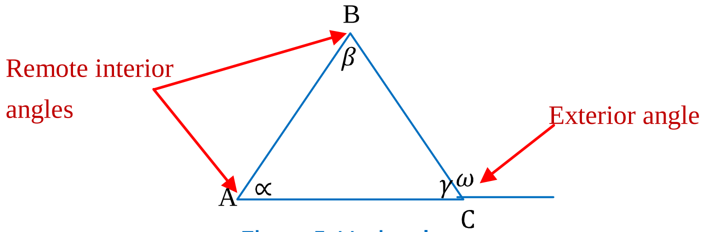


Figure 5.44 triangle

Statements	Reasons
1. $\gamma + \omega = 180°$ 2. $\alpha + \beta + \gamma = 180^o$ 3. $\alpha + \beta + \gamma = \gamma + \omega$ 4. $\alpha + \beta = \omega$	1. Supplementary angles 2. Angle sum theorem 3. Substitution 4. Subtracting γ from both sides

Example 9. Calculate the value of the variables in Figures 5.45 below.

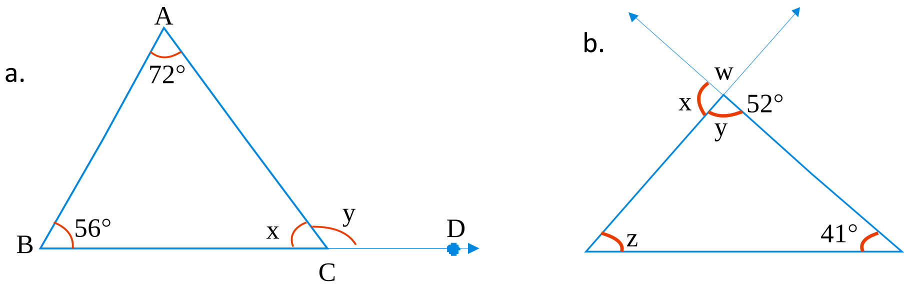


Figure 5.45

Solution:

a. m(∠ABC) + m(∠BCA) + m(∠CAB) = 180^o …….. Angle sum theorem.

$56^o + x + 72^o = 180^o$ ………. Substitution.

$x + 128^o = 180^o$

$x = 180^o - 128^o$

$x = 52^o$

Now m(∠ABC) + m(∠BAC) = m(∠ACD) …….. Theorem 5.5.

$56^o + 72^o = y$ ……… Substitution.

$128^o = y$

Or $y = 128^o$

b. $y + x + w + 52^o = 360^o$

$y + 52 + w + 52 = 360^o$ …………… $x = 52^o$ measures of vertically. opposite angles

$y + w + 104 = 360^o$

$y + y = 256^o$ ……… $W = y$ (measures of vertically opposite. angles are equal) and substitution.

$2y = 256^o$

$y = 128^o$

Thus $z + y + 41^o = 180^o$ ………. Angle sum theorem.

$z + 128^o + 41^o = 180^o$ ………… Substitution.

$z = 180^o – 169^o$

$z = 11^o$

Exercise 5F

1. Find the degree measures of marked angles in Figure 5.46 below (the letters a – h represent degree measures of the angles).

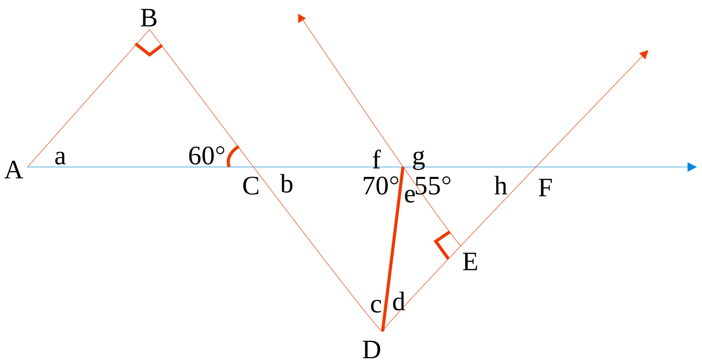


Figure 5.46

2. Find the degree measures of the marked angles in Figure 5.47 below.

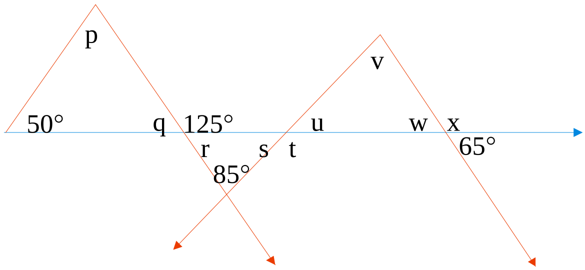


Figure 5.47

3. Find the degree measure β of the marked angles in Figure 5.48 below.

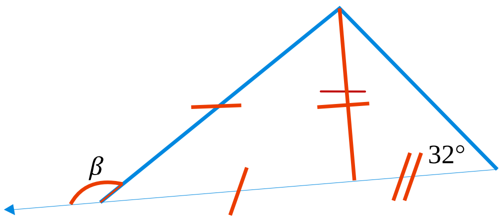


Figure 5.48

4. In Figure 5.49 to the right if m(∠ADB) = 70° and m(∠BCA)= 30°, then what is m(∠CBD)?

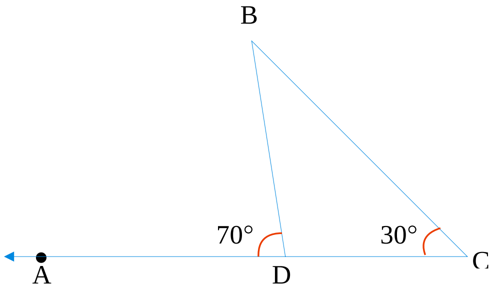


Figure 5.49

5. In Figure 5.50 given to the right . What is the sum of the measures a, b, c,d, e and f of the angles marked.

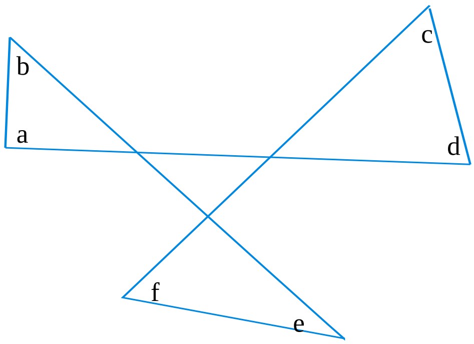


Figure 5.50

Challenge Problem

6. In Figure 5.51 given to the right, m(∠ABC) = 32°, m(∠BHE) = 42° and m(∠ADE) = 48°. Find m(∠NAD).

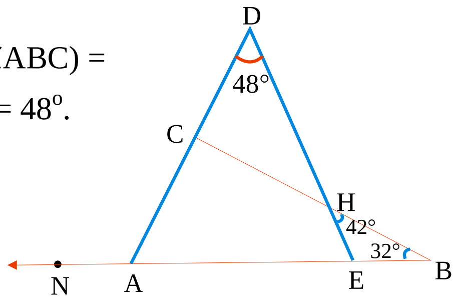


Figure 5.51

7. In Figure 5.52 given to the right $\overline{DE} \parallel \overline{AB}$, $m(\angle D) = 42^o$, $m(\angle BCA) = 108^o$. Find $m(\angle B)$ and $m(\angle A)$.

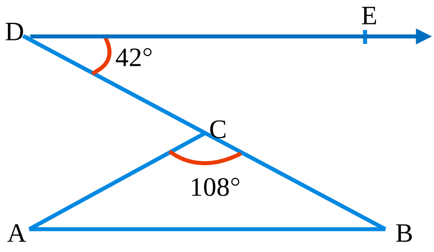


Figure 5.52

A. The sum of the interior angles of a polygon

Activity 5.7

1. Calculate x.

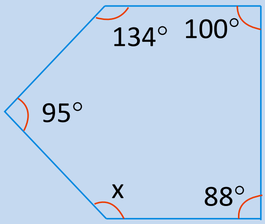


Figure 5.53 pentagon

2. Calculate y

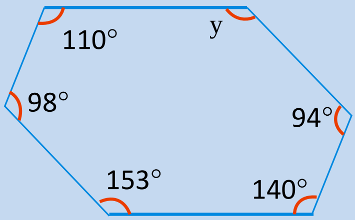


Figure 5.54 hexagon

3. Calculate the measure of the interior angles of:

a. a square

b. a pentagon

c. a hexagon

d. a heptagon

The measures of all interior angles of a quadrilateral always add up to 360^o.

i) You can see this by checking that the angles in this quadrilaterial add up to 360^o or

$m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360^o$

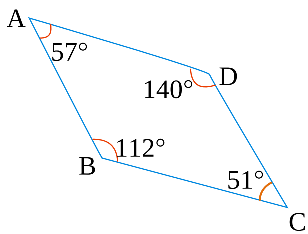


Figure 5.55 Quadrilateral

ii) By dividing the quadrilateral in to two triangles so that the measures of the interior angles of the two triangles add up to $180^{o} + 180^{o} = 360^{o}$.

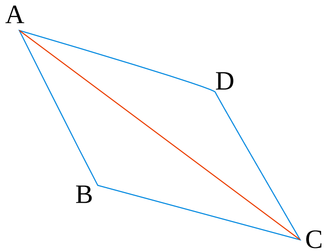


Figure 5.56 Quadrilateral

iii) By tearing off the four corners and put the angles together. They make a full trun of 360^{o}.

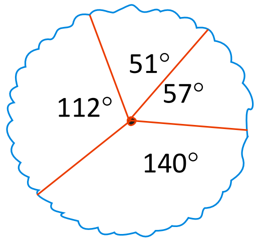


Figure 5.57

If you draw all the diagonals from one vertex of a convex polygon, you will find non–overlapping triangles and you can also find that the sum of the measures of the interior angles of the polygon by adding the measures of all interior angles of these triangles in the polygon. Look at Figure 5.58 and count the triangles formed triangles in each polygon. Apply the angle sum theorem triangles in each polygon and try to find the sum of the measures of all the interior angles of each polygon.

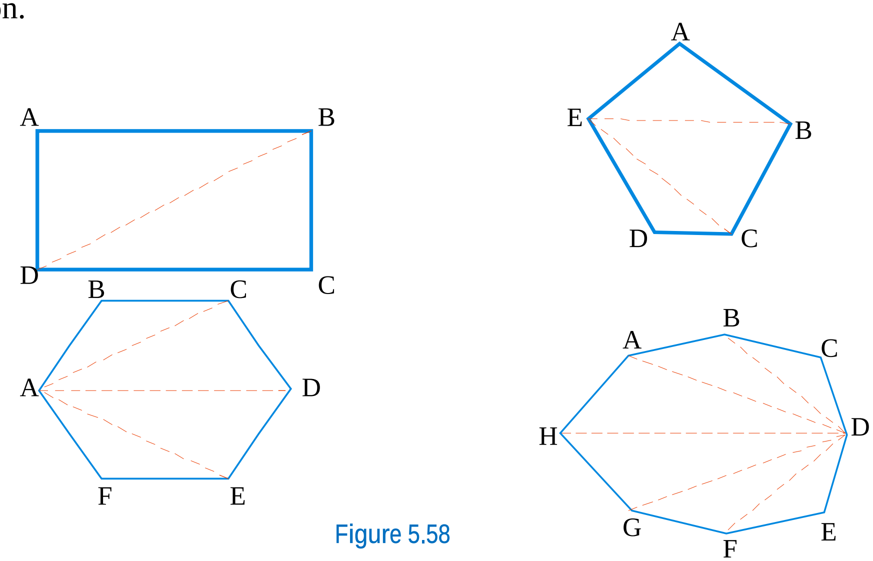


Figure 5.58

?

Can you find a formula which will help you to find the sum of the measures of all the interior angles of any given convex polygon?

Example 10. In a pentagon, 3 triangles can be formed by the diagonals from one vertex see in Figure 5.59 below. (The letters represent the degree measures of the angles).

By the angle sum Theorem,

$a + k + h = b + g + f = c + e + d = 180^o$

Let the sum of the interior angles of the pentagon be β

Then $\beta = a + b + c + d + e + f + g + h + k$

$\beta=(a+k+h)+ (b+g+f)+ (e+d+c)$

$\beta = 180^o + 180^o + 180^o$

$\beta = 3 \times 180^o$

Hence $\beta = 540^o$.

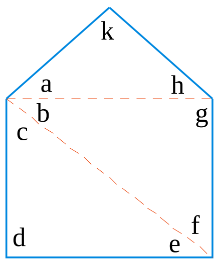


Figure 5.59pentagon

Example 11. In a hexagon, 4 triangles can be formed by the diagonals from one vertex in Figure 5.58.

By the angle sum theorem:

$a + b + c = d + k + j = e + g + i = f + \ell + h = 180^o$

Let the sum of the interior angles of the hexagon be β.

Then $\beta = a + b + c + d + e + f + g + h + i + j + k + \ell$

$\beta= (a + b + c) + (d + j + k) + (e + g + i) + (f + h + \ell)$

$\beta = 180^o + 180^o + 180^o + 180^o$

$\beta = 4 \times 180^o$

Hence $\beta = 720^o$

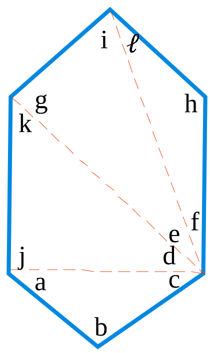


Figure 5.60 Hexagon

Table 5.3 Number of sides of a polygon and the respective sum of degree measures of all its interior angles.

Number of sides of polygon	Number of triangles formed by diagonals from one vertex	Sum of degree measures of interior angles
3	1	$1 \times 180^o = 180^o$
4	2	$2 \times 180^o = 360^o$
5	3	$3 \times 180^o = 540^o$

6	4	$4 \times 180^o = 720^o$
7	5	$5 \times 180^o = 900^o$
8	6	?
9	7	?
10	8	?
n	n-2	?

You might have noticed that for an n – sided polygon the number of triangles formed is 2 less than the number of sides n. If that is so you can write the following:

The formula for the number of triangle, T, determined by diagonals drawn from one vertex of an n - sided polygon is $T = n - 2$.

? What did you notice again?

Since you have already seen that the sum of the measures of the three angles of a triangles is 180^o, you can make the following generalization.

The formula for the sum, S of the measures of all the interior angles of a polygon of n sides is given by $S = (n - 2)\ 180^o$.

Definition 5.13: A polygon whose all sides are congruent is called an Equilateral polygon.

An Equilateral triangle and a rhombus are examples of equilateral polygons.

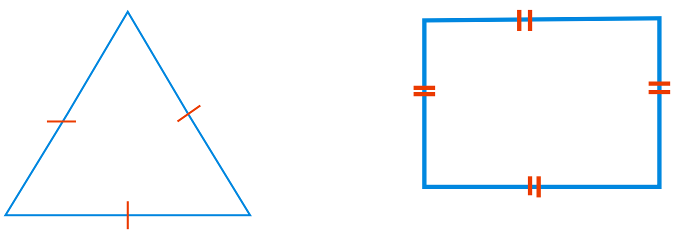


Figure 5.61 Examples of equilateral polygons

Definition 5.14: Apolygon whose all angles are congruent (of the same size or measure) is called an equiangular polygon.

A rectangle is an example of equiangular polygon.

Definition 5.15: A polygon which is both equilateral and equiangular is called a regular polygon.

Equilateral triangle and square are examples of regular polygon.

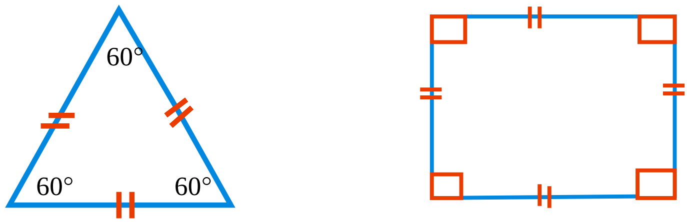


Figure 5.62 Example of regular polygon.

Example 12. Find the sum of the measures of all the interior angles in a polygon having 30 sides.

Solution:

$n = 30$

$S = (n-2) \times 180^o$ ……… Given formula

$S = (30-2) \times 180^o$ ………… Substitution

$S = 28 \times 180^o$ ……………… Simplifying

$S = 5040^o$

Therefore; the sum S, of the measures of all the angles of the polygon is 5040^o.

Example 13. If all the angles of a polygon with 40 sides are congruent, then find the measure of each angle of the polygon.

Solution:

$n = 40$

let y be the measure of each angle of the polygon.

Then the sum of the angles of the polygon on one hand is:

S = 40y ………. Equation 1

On the other hand, the sum of the angles is given by the formula:

S = (n – 2) × 180^o …….. Equation 2

Equating equation(1) and Equation(2) we get:

$40y = (n-2) \times 180^o$

$40y = (40-2) \times 180^o$

$40y = 38 \times 180^o$

$y = \frac{38 \times 180^o}{40}$

$y = 171^o$

so each of the 40 sides has a measure of 171^o .

Example 14. The angles of a hexagon are x, $2\frac{1}{2}x, 3\frac{1}{2}x$, 2x, x and 2x. what is the value of x.

Solution:

The sum of the measures of the interior angles of a hexagon is 720^o.

Thus, $x + 2\frac{1}{2}x + 3\frac{1}{2}x + 2x + x + 2x = 720°$

$12x = 720^o$

$x = \frac{720^o}{12}$

$x = 60^o$

Example 15. The angles of a pentangon are x, (x + 20°), (x – 15°), 2x and $\left(\frac{3}{2}x + 30^o\right)$. Find the value x.

Solution: For a pentagon the sum of the measures of the interior angles is 540^o

Thus $x + (x + 20^o) + (x - 15^o) + 2x + \left(\frac{3}{2}x + 30^o\right) = 540^o$

$x + x + 20^o + x - 15^o + 2x + \frac{3}{2}x + 30^o = 540^o$

$\frac{13}{2}x + 35^o = 540^o$

$\frac{13}{2}x = 505^o$

$13x = 1010^o$

$x = \frac{1010^o}{13}$

Therefore, the value of x is $\frac{1010^o}{13}$

Note: The measure of each interior angle of an n-sided regular polygon is $\frac{(n-2)\times 180^o}{n}$.

Example 16. Find the degree measure of each interior angle of a regular,

a. 6 – sided polygon.

b. 18-sided polygon.

Solution:

a. The degree measure of a regular polygon $= \frac{(n-2)\times 180^o}{n}$

$= \frac{(6-2)\times 180^o}{6}$ …….. For n = 6

$= \frac{4 \times 180^o}{6}$

$= 120^o$

b. The degree measure of a regular polygon $= \frac{(n-2)\times 180^o}{n}$

$= \frac{(18-2)\times 180^o}{18}$ …….. For n = 18

$= 16 \times 10^o$

$= 160^o$

Example 17. If the sum of the measures of all the interior angles of a polygon is 1440^o, how many sides does the polygon have?

Given: $S = 1440^o$

Required: let n = the number of sides of the polygon?

Solution: $S = (n-2) \times 180^o$ …………… Given formula

$(n-2) \times 180^o = 1440^o$ ………. Substitution

$n - 2 = \frac{1440^o}{180^o}$

$n - 2 = 8$

$n = 10$

So, the polygon has 10 sides.

Exercise 5G

1. Find the sum of the measures of all the interior angles of a polygon with the following number of sides.
   a. 12 b. 20 c. 14 d. 11
2. Find the sum of the measures of each interior angles of:
   a. a regular pentagon. c. a regular hexagon.
   b. a regular octagon. d. a regular 15 sided figure.
3. Can a regular polygon have an interior angle of:
   a. 160^o ? b. 135^o? c. 169^o? d. 150^o?
   Exaplain why?
4. An octagon has angles of 120^o, 140^o, 170^o and $165°$. The other angles are all equal. Find their measures.
5. The measures angles of a hexagon are 4x, 5x, 6x, 7x, 8x and 9x. Calculate the size of the largest angle.
6. The angles of a pentagon are 6x, $(2x + 20^o)$, $(3x – 20^o)$, 2x and 14x. Find x.
7. The interior angle of a polygon is $100°$. The other interior angles are all equal to $110°$. How many sides has the polygon?
8. In Figure 5.63 to the right, what is the sum of the measures of angles given by a, b, c, d, e and f.

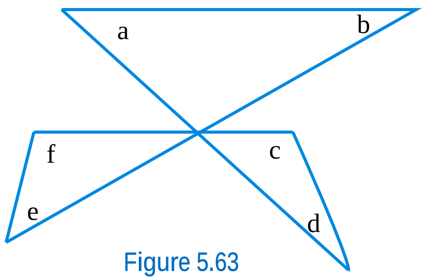


Figure 5.63

9. In Figure 5.64 to the right, what is the sum of the measures of angles given by a, b, c, d, e, f, g, and h.

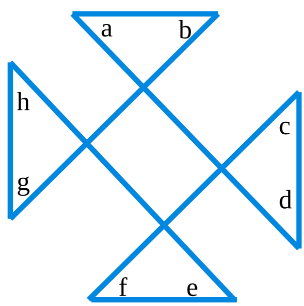


Figure 5.64

10. Find the values of β, θ, σ, ∝, and δ.

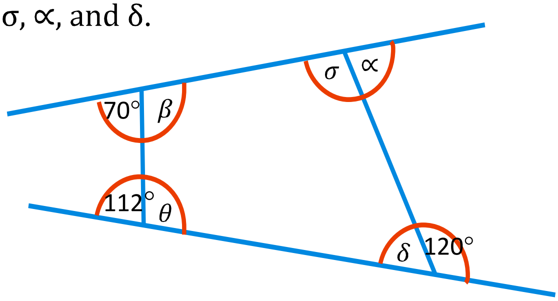


Figure 5.65

Challenge Problem

11. In Figure 5.66 shown, prove that the sum of all the interior angles is equal to two right angles.

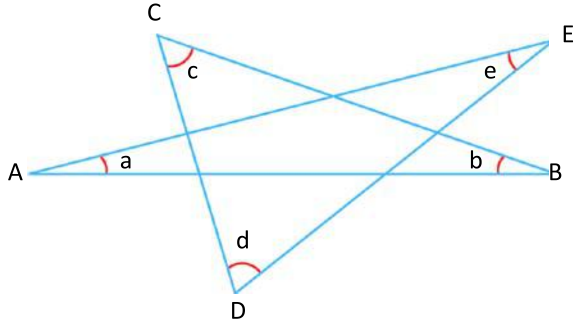


Figure 5.66

12. In Figure 5.67 given to the right, what is the sum of the measure of angles given by a, b, c, d, e, f, g, h and i?

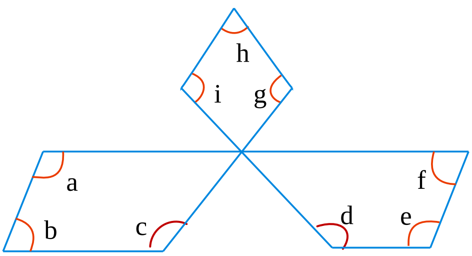


Figure 5.67

13. In Figure 5.68 given to the right, what is the measures of angles given by a, b, c, d, e, f, g, h, i and j.

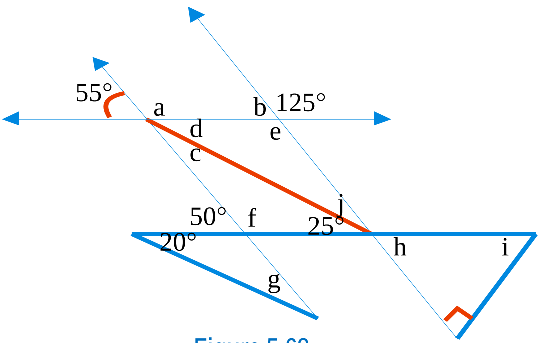


Figure 5.68

5.3. Measurement

There are only three special polygons, other than the rectangle (square) whose areas are considered important enough to investigate. These polygons are the triangles, the parallelogram and trapezium. The area of any other polygon is found by drawing segments as to divide it into a combination of these four figures.

5.3.1. Area of a Triangle

Group Work 5.4

Discuss with your friends.

1. The perimeter of a square is 64cm. what is the length of a side?
2. The area of a square is $81cm^2$. What is its perimeter?
3. In a rectangle the length is twice the width. The perimeter is 36cm. Find the length, width and area of this rectangle.
4. In a rectangle the length is 20cm more than the width. The perimeter is 140cm. Find the area.
5. Suggest units of area to measure the area of the following regions (Choose from mm^2, cm^2, m^2 or km^2).
   a. the page of an exercise book.
   b. the floor of your class room.
   c. a television screen.

The area of a triangle tells us how many unit squares the triangle contains. To find the area of a triangle, you need to know the base and the height of the triangle.

From grade six mathematics lessons you remember that triangles were classified according to the lengths of their sides and the sizes of their angles.

? Do you remember what they are called.?(what are they)?

Write the names of the triangles given below.

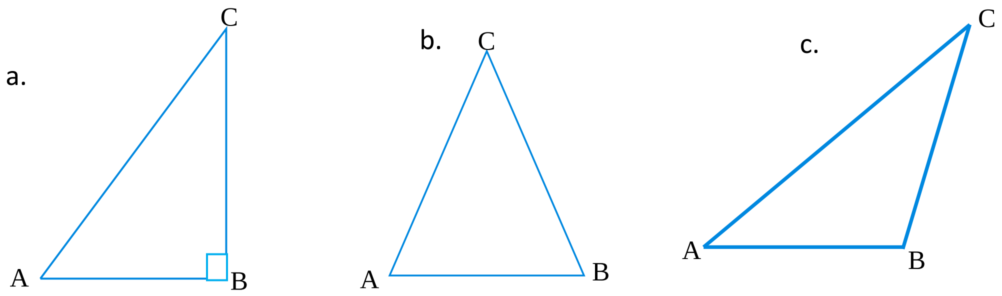


Figure 5.69 Types of triangles

You will see how the area of each triangle given above shall be computed.

First you will revise on the area of a right triangle. To compute the area of such types of figures you will apply the knowledge of the area of rectangles. (see Figure 5.70). You already know that if the sides of a triangle are ‘a’ and ‘b’ then the area A of the rectangle is given by:

$A = a \times b$. We also know that each diagonal divides the rectangle in to two congruent triangles: Hence, the area A of the right-triangle ABC is given by $A = \frac{a \times b}{2}$

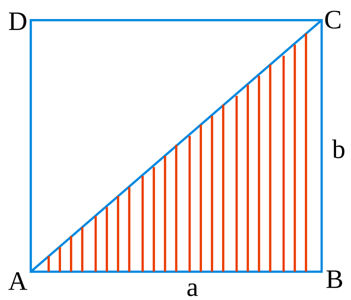


Figure 5.70 rectangle

Note: In the rectangle ABCD shown in Figure 5.70 above the sides $\overline{AB}$ and $\overline{BC}$ of triangle ABC are respectively called the base and the height or altitutde.

Theorem 5.6: The area of a right – angled triangle ABC with base b and height h is given by

$$A = \frac{bh}{2}$$

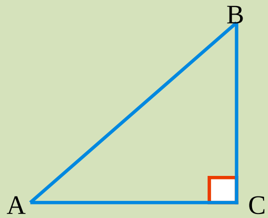


Figure 5.71 Right – angled triangle

? How can you find the formula for the area of a triangle?

Now you will see how the area of a triangle shall be computed. For this you are going to use the knowledge you have acquired before. Consider the following two triangle

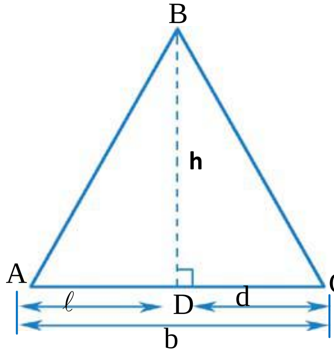


Figure 5.72 Acute angled triangle

You know that in Figure 5.72 the altitude/height/ divides the triangle in to two right triangles.

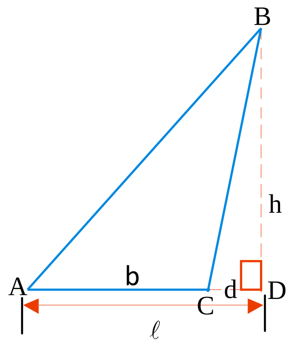


Figure 5.73 Obtuse angled triangle

Hence, the a(ΔABC) = a(Δ ABD) – a(ΔCBD)

$$= \frac{1}{2}\ \ell h - \frac{1}{2}dh$$

$$= \frac{1}{2}h\ (\ell\ - d)\ \ldots\ldots\ \text{Note thet, } \ell - d = b$$

$$= \frac{1}{2}bh$$

Similarly consider in Figure 5.73

Hence, the a(ΔACB) = a(ΔABD) – a(ΔCBD)

$$= \frac{1}{2}\ell h - \frac{1}{2}dh$$

$$= \frac{1}{2}h(\ell\text{-}d)\ldots\ldots\ldots\ \text{Note that, } \ell - d = b$$

$$= \frac{1}{2}bh$$

Theorem 5.7: The area A of a triangle whose base is b and altitude to this base is h is given by $A = \frac{1}{2}bh$

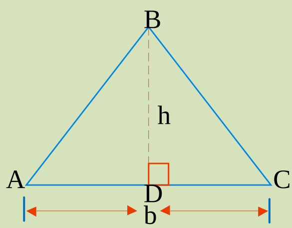
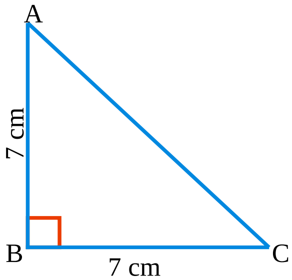


Figure 5.74

Example 18. Find the area of an isosceles right angled triangle with length of legs 7cm.

Solution: See the following figure below



ΔABC is isosceles right angled triangle, with length of legs AB = BC = 7cm

Thus, $a(\Delta ABC) = \frac{1}{2}(AB \times BC)$

$= \frac{1}{2}(7cm \times 7cm)$

$= \frac{49}{2}\,cm^2$

Therefore, the area of isosceles right angled triangle is $\frac{49}{2}\,cm^2$.

Example 19. In Figure 5.75 to the right, the outer triangle has base 8cm and height 7cm.

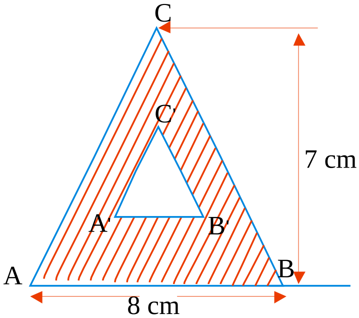


Figure 5.75

a. Calculate the area of the outer triangle. The base and height of the inner triangle are half those of the outer triangle.

b. Calculate the area of the inner triangle.

c. Calculate the area of the shaded part(region).

Solution:

a. $a(\Delta ABC) = \frac{1}{2}bh$ .... Theorem 5.7

$= \frac{1}{2}(8cm \times 7cm)$......... Substitution

$= 28\ cm^2$ .... Simplifying

b. $a(\Delta A'B'C') = \frac{1}{2}\left(\frac{bh}{2}\right)$

$= \frac{1}{2}\left(4cm \times \frac{7}{2}cm\right)$

$= 7cm^2$

c. a(shaded region) = a (outer triangle) – a(inner triangle)

$= 28cm^2 - 7cm^2$

$= 21cm^2$

Example 20. In Figure 5.76 below $\overline{CD} \perp \overline{AB}$ with AB = 12cm and if the vertex C is moved to E by 3cm, then what is the area of the shaded region?

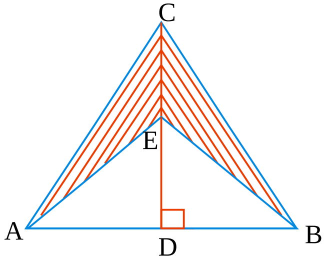


Figure 5.76

Solution:

Let DE = x cm, then

Area of shaded region

$= a(\Delta ABC) - a(\Delta ABE)$

$= \frac{1}{2}(12(x+3)) - \frac{1}{2}(12(x))$

$= 6x + 18 - 6x$

$= 18$

Therefore, the area of the shaded region is $18cm^2$.

Example 21. Find the area of the shaded part of the Figure 5.77 given below.

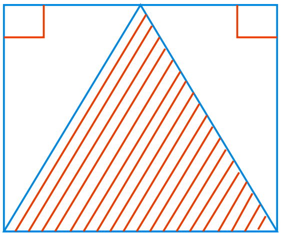


Figure 5.77

Solution:

$a \text{ (shaded part)} = \frac{1}{2}bh$

$= \frac{1}{2}(60cm \times 20cm)$

$= 600 \text{ cm}^2$

Therefore, the area of the shaded region is $600cm^2$.

Note: If the lengths of the sides of a triangle are a, b and c, then the perimeter p of the triangle is $p = a + b + c$.

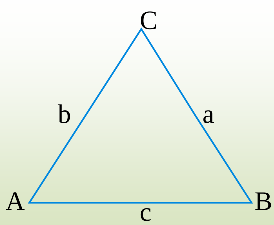


Figure 5.78

Example 22. If the perimeter of the isosceles triangle ABC shown in Figure 5.79is 14cm and its base side is 6cm, what is the length of its equal sides?

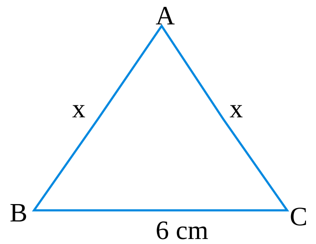


Figure 5.79

Solution:

Let x = the length of the equal sides(in cm)

Since the perimeter of a triangle is the sum of the lengths of its side,

Then x + x + 6 = P

$$2x + 6 = 14$$

$$2x = 8$$

$$x = 4$$

Thus, the lengths of its equal sides are 4cm each.

Exercise 5H

1. Find the area of each triangles.

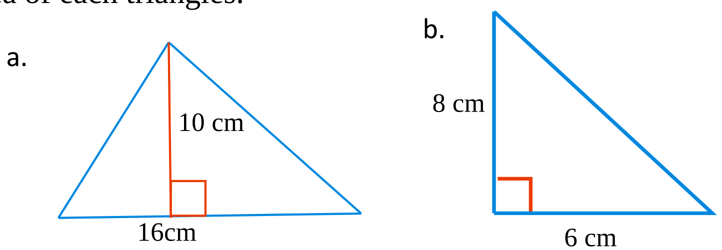


Figure 5.80

2. In Figure 5.81 represents a wall of a certain building. Find the area of the wall.

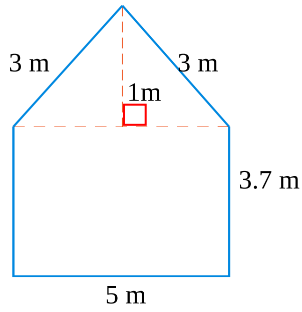


Figure 5.81

3. What is the area of the triangle?

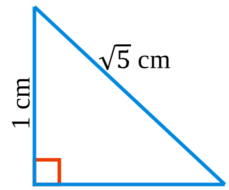


Figure 5.82

4. In Figure 5.83, what is the area of the shaded part of the rectangle?

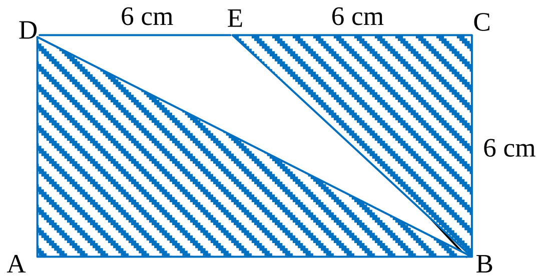


Figure 5.83

5. What is the perimeter and area of to the right Figure 5.84.

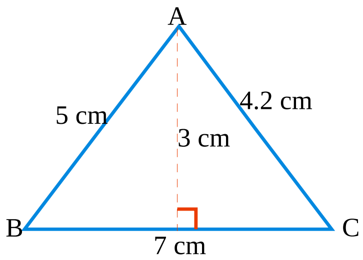


Figure 5.84

Challenge Problem

6. In Figure 5.85, EFN is a straight line. Find the area of ΔDEF.

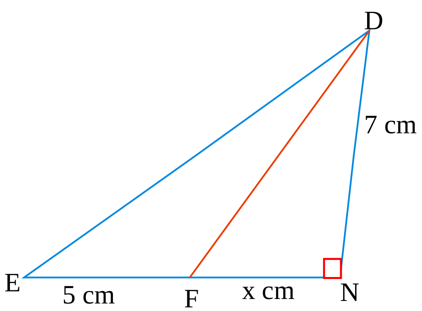


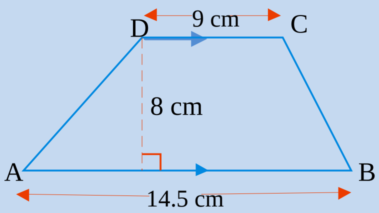
Figure 5.85

5.3.2. Perimeter and Area of Trapezium

Group Work 5.5

Discuss with your friends

1. Find the perimeter and area of a trapezium if its parallel sides are 24cm and 48cm, and its non parallel sides are each 13cm long.
2. Calculate the areas of each trapezium given below.



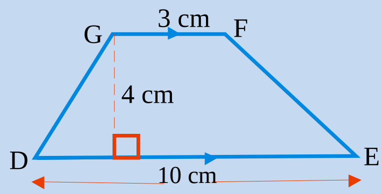


Figure 5.86

3. The area of a trapezium is $276cm^2$. The altitude is 12cm and one base is 14cm long. Find the other base.

? How can you find the formula for the area of a trapezium?

Consider the trapezium ABCD shown in Figure 5.87 below.

Now divide the trapezium in to two triangles, namely ΔABC and ΔACD. These two triangles have the same altitude h, but different bases b_1 and b_2. Where b_1 and b_2 are the lengths of the parallel sides and h is the perpendicular distance between them.

Thus $a(ABCD) = a(\Delta ABC) + a(\Delta ACD)$

$$= \frac{1}{2}(BC \times AF) + \frac{1}{2}(AD \times CE)$$

$$= \frac{1}{2}(b_1h) + \frac{1}{2}(b_2h) \quad \ldots AF = CE = h$$

$$= \frac{h}{2}(b_1 + b_2)$$

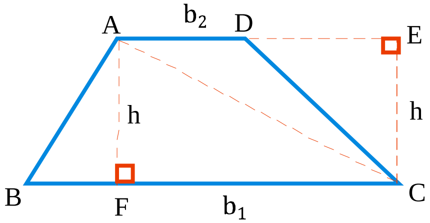


Figure 5.87

Therefore, the area of the trapezium is $\frac{h}{2}(b_1 + b_2)$.

Theorem 5.8: If the lengths of the bases of a trapezium are denoted by b_1 and b_2 and its altitude is denoted by h, then the area A of the trapezium is given by:

$$A = \frac{h}{2}(b_1 + b_2).$$

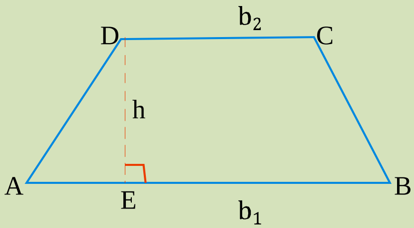


Figure 5.88

Example 23. What is the area of the following trapezium Shown in Figure 5.89.

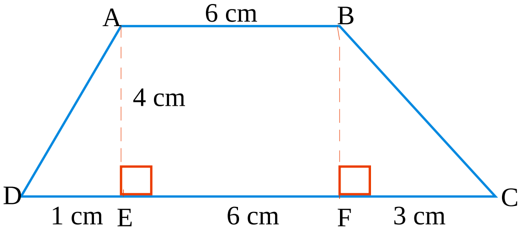


Figure 5.89

Solution:

Let $b_1 = 6\text{cm}$, $b_2 = 10\text{cm}$ and $h = 4\text{cm}$

Then $A = \frac{h}{2}(b_1 + b_2)$

$A = \frac{4\text{cm}}{2}(6\text{cm} + 10\text{cm})$

$A = 2\text{cm}\,(16\text{cm})$

$A = 32\text{cm}^2$

Therefore, the area of the trapezium is 32cm^2.

Note: If the length of the sides of a trapezium ABCD are a, b, c and d, then the perimeter P of the trapezium is given by:

$P = AB + BC + CD + DA$

$P = a + b + c + d$

Example 24. In Figure 5.90 to the right, find the perimeter of the trapezium ABCD.

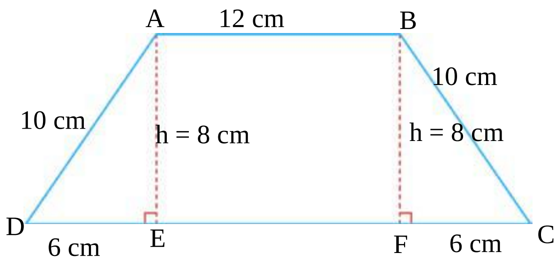


Figure 5.90

Thus P (trapezium ABCD) = AB + BC + CD + DA …….. Perimeter

= 12 cm + 10cm + 24cm + 10cm

= 56 cm

Therefore, the perimeter of the trapezium ABCD is 56 cm.

Exercise 5I

1. Find the perimeter of the trapezium in Figure 5.91 if x = 9 and y = 7.

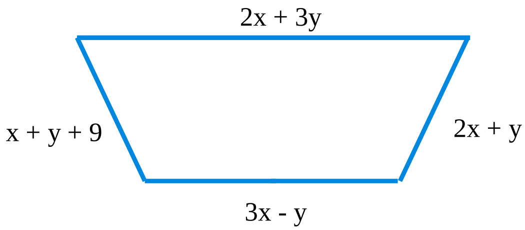


Figure 5.91

2. The area of a trapezium is $35cm^2$. Find its altitude if the beses are 6cm and 8cm.
3. If the area of the trapezium ABCD is $30cm^2$, find the value of b_1. (see Figure 5.92).

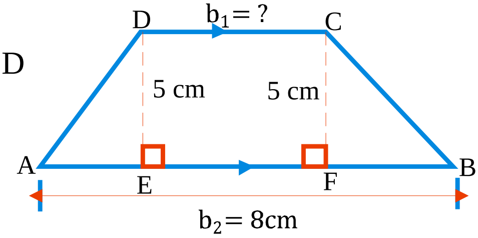


Figure 5.92

5.3.3. Perimeter and area of Parallelogram

Activity 5.8

1. Find the area A of parallelogram PQRS where b = 10cm and h = 6.7cm.

2. Derive the area formula for a parallelogram.

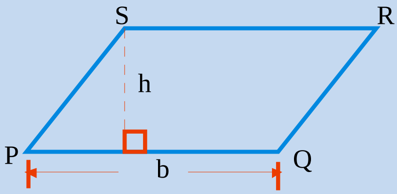


Figure 5.93

Now you pay attention to how the area of a parallelogram shall be computed, in doing so you are going to use the knowledge you have acquired so far. Let us first look at Figure 5.94 to the right.

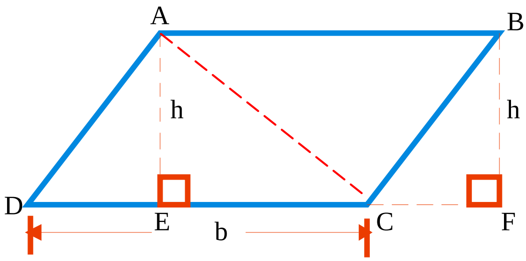


Figure 5.94

You know that the diagonal divides the parallelogram in to two congruent triangles.

Hence, the a(ABCD) = a (ΔADC) + a(ΔABC)

$$= \frac{1}{2}(DC \times AE) + \frac{1}{2}(AB \times BF)$$

$$= \frac{1}{2}\,bh + \frac{1}{2}bh$$ ………. DC = AB = b because opposite sides of a parallelogram have equal length

$$= \frac{2}{2}bh$$

$$= bh$$

Therefore the area of the parallelogram = length of base × the perpendicular height between this base and its opposite side.

Theorem 5.9: The area A of a parallelogram with length of base b and corresponding height h is given by A = bh.

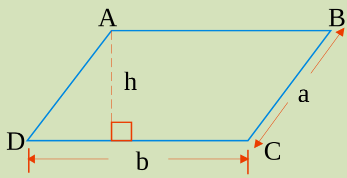


Figure 5.95

Example 25. The area of a parallelogram is $48cm^2$. Find its altitude if the base is 6cm.

Solution: A($_{\text{parallelogram}}$) = bh ……………….Theorem 5.9

$48cm^2 = 6cm \times h$ ……….. Substitution

Then $h = \frac{48cm^2}{6cm}$ ……………… Dividing both sides by 6

$h = 8cm$

Therefore, the height of the parallelogram is 8cm.

Note: In Figure 5.96, if the length of the sides of a parallelogram ABCD are a and b, then the perimeter P of the parallelogram is given by:

P = AB + BC + CD + DA

P = b + a + b + a

P = 2a + 2b

P = 2(a + b)

Example 26. The perimeter of the parallelogram is 46 cm. Find the sum of the lengths of its sides.

Solution: Let a and b be the length of sides of the parallelogram.

Then $P = 2(a + b)$

$46 \text{ cm} = 2(a + b)$

$a + b = 23 \text{ cm}$

Therefore, the sum of the lengths of sides is 23cm.

Exercise 5J

1. In Figure 5.96, AP, AQ are altitudes of the parallelogram ABCD.
   a. If AQ = 4cm , CD = 5cm, find the area of ABCD.
   b. If the area of ABCD = 24cm^2, AB = 6cm then find AQ.
   c. If AB = 5cm, AP = 4cm, AD = 6cm, then find AQ.

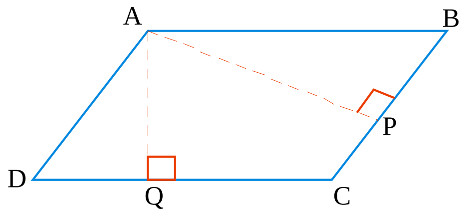


Figure 5.96

2. PQRS is a parallelogram of area 18cm^2. Find the length of the corresponding altitudes if PQ = 5cm and QR = 4cm.

3. ABCD is a parallelogram in which AB = 3cm, BC = 12cm and the perpendicular from B to AD is 2.5cm. Find the length of the perpendicular from A to CD.

\

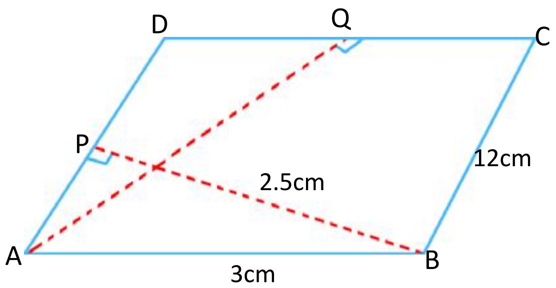


Figure 5.97

Challenge Problem

4. The lengths of the two altitudes of a parallelogram are 4cm, 6cm and the perimeter of the parallelogram is 40cm. Find the lengths of the sides of the parallelogdram.

5.3.4. Circumference of a Circle

Activity 5.9

Discuss with your teacher before starting the lesson under this topic you will need a ruler, pair of compasses and some string or thread.

1. a. Mark a point on your paper. Use your compasses and draw three circles of different radii (or diameters) with the marked point as a center.
   b. put the thread slowly and carefully around each circle untill its both ends join (do not over lap).
   c. stratch this thread against the ruler and measure its length which gives the circumference.
   d. calculate the ratio = $\frac{\text{circumfrence of the circle}}{\text{Diameter of the circle}}$.
2. What do you notice about the ratio = $\frac{\text{circumference of a circle}}{\text{diameter of this circle}}$.

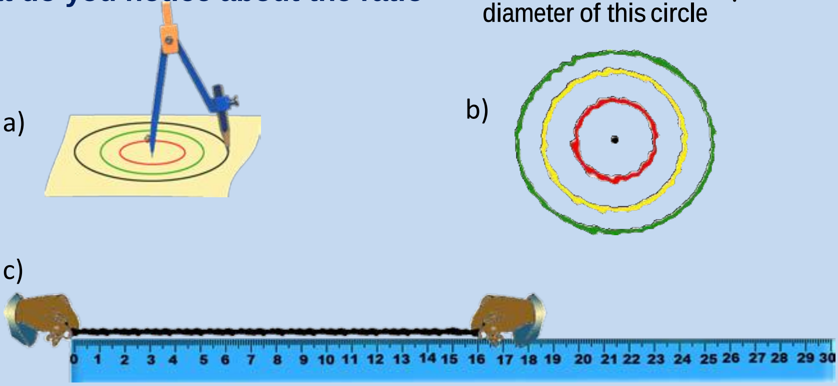


Figure 5.98

3. With your own words describe each of the following
   a. center of a circle.   b. radius of a circle.   c diameter of a circle.

The perimeter of a circle is called its **circumference**. The circumference of a circle is related to its radius or diameter.

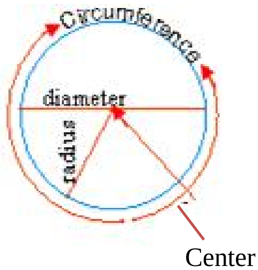


Figure 5.99

Introducing π(pi)

If you compare the answers to the class Acitivity 5.9 with that of your friends, you should find that for each circle the circumference of a circle divide by its diameter is approximately equal to 3.14. The actual value is a special number represented by π.

You can not write the exact value of π, because the number π is a non – recurring or non-terminating decimal which is found some where between 3.141592 and 3.141593.

If you press the π key on a calculator the value 3.141592654… appears. In calculations we often use the value of π correct to two decimal place as 3.14 or correct to three decimal place as 3.142 or $\frac{22}{7}$.

Finding a formula for the circumference

Representing the circumference by 'c' and the diameter by 'd', you can equate the ratio in Activity 5.9 (1d) as $\frac{\text{circumference}}{\text{diameter}} = \frac{C}{d} \approx \frac{22}{7} \approx 3.14 \approx \pi$.

To give a formula to find the circumference of a circle using its diameter. Thus,

$\frac{c}{d} \times d = \pi \times d$ ……….. Multiplying both sides by d

So $C = \pi d$ ……….. The required formula

You can also rewrite the formula for the circumference using the radius.

Since the diameter is twice the radius: $d = 2r$

So $C = \pi \times 2r$

Or $C = 2\pi r$

Theorem 5.10: The circumference of a circle whose diameter d is given by:

$C = \pi \times d$ or

$C = \pi \times 2r$ where c is the circumference

d is the diameter

r is the radius

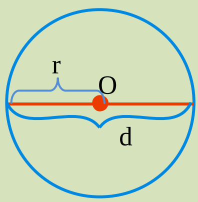


Figure 5.100

Example 27. Find the circumference of a circle with diameter 6cm.
(Hint π = 3.14) .

Solution: $C = \pi d$

$C = \pi \times 6cm = 3.14 \times 6cm = 18.84cm$

Therefore, the circumference of a circle is 18.84cm.

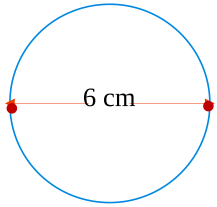


Figure 5.101

Example 28. Find the circumference of a circle with radius 5cm.

Solution: $C = 2\pi r$

$C = 2 \times 3.14 \times 5cm$

$C = 31.4$ cm

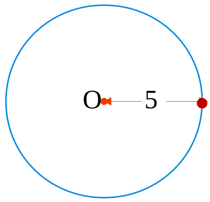


Figure 5.102

Exercise 5K

1. Find the circumference of the circle with each of the givn diameters below. Write your answers to three significant digits. (Take π =3.14)
   a. 4cm   c. 8cm   e. 2.5cm
   b. 10cm   d. 12cm   f. 8.25cm
2. Find the circumferences of the circles with the radii given below. Write your answers to three significant digits (Take π = 3.14).
   a. 8cm   c. 12cm   e. 3.6cm
   b. 50cm   d. 2.5cm   f. 8.26cm
3. Ahmed's bike wheel has a circumference of 125.6cm. Find the diameter and the radius of the wheel.
4. A piece of land has a shape of semicircular region as shown in Figure 5.103 to the right . If the distance between points A and B is 200 meteres, find the perimeter of the land.

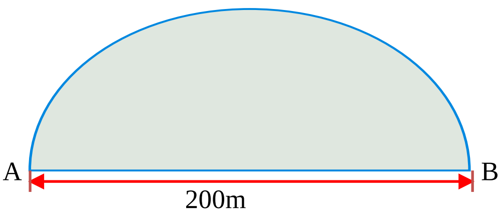


Figure 5.103

5. Find the perimeter of the field whose shape is as shown in Figure 5.104 to the right. The arcs on the left and right are semicircle of radius 100m and the distance between pairs of end points of the two arcs is equal to 200 m each.

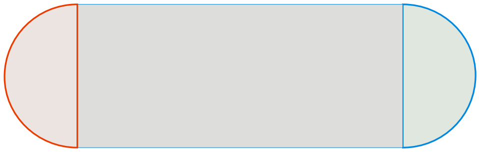


Figure 5.104

Challenge Problem

6. In Figure 5.105 to the right find the perimeter of the quarter circle.

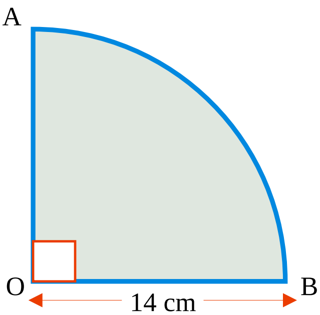


Figure 5.105

5.3.5. Area of a circle

Activity 5.10

1. **Find the areas of the circles with radius:**

   a. 8cm **b. 5cm** **c. 10cm** **d. 12cm**

2. **Find the areas of the circles with these diameters:**

   a. 18cm **b. 20cm** **c. 16cm** **d. 17cm**

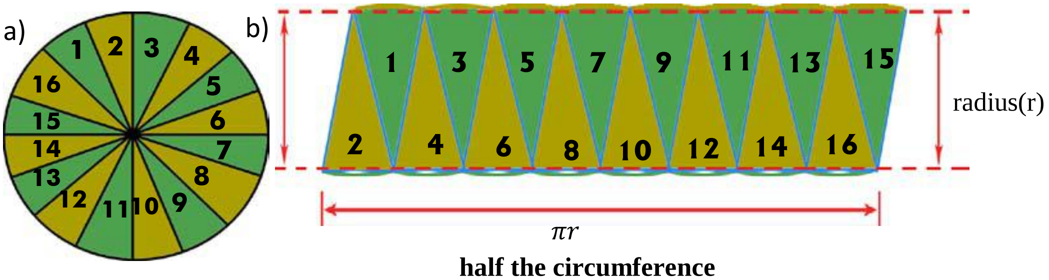
As you have seen, in the previous lessons, the area of a plane figure can be determined by counting unit squares fully contained by the figure. You have seen this when you were discussing about the area of a rectangle. In this lesson you will learn how to find a formula for finding the area of a circle. To find the formula for the area of a circle the following steps is very important.

Step i: Draw a circle with radius 4cm.

Step ii: Draw diameters at angles of 20^{o} to each other at the center to divide the circle in to 16 equal parts. Carefully cut out these 16 parts.

Step iii: Draw a straight line. Place the cut – out pieces alternately corner to curved edge against the line. Stick them together side by side and close enough.

It would be very difficult to cut out the parts of the circle if you used 1^o between the diameters, but the final shape whith the same color shade the sectors that are labelled by odd numbers as shown would be almost an exact rectangle.



half the circumference

Figure 5.106

In Figure 5.106 (b) the two longer sides of the rectangle make up the whole circumference πd or 2πr, so one length is πr. The width is the same as the radius of the circle, r.

So the area of the rectangle = length × width

$$= \pi r \times r$$

$$= \pi r^2$$

This is the same as the area of the circle, so

$$\boxed{\text{Area of acircle} = A = \pi r^2}$$

Theorem 5.11: The area of a circle whose radius r unit long is given by

$$A = \pi r^2 \text{ or } A = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4} \text{ since } \frac{d}{2} = r$$

Example 29. Find the area of a circle with diameter 8cm.

Solution: $A = \frac{\pi d^2}{4}$

$A = \frac{\pi}{4}(8\text{cm})^2$

$A = \frac{\pi}{4} \times 64\text{cm}^2$

$A = 16\pi$ cm^2. Therefore, the area of a circle is 16πcm^2.

Example 30. In Figure 5.107 to the right , find the area of the quarter circle of radius 8 cm. (use $\pi = 3.14$) .

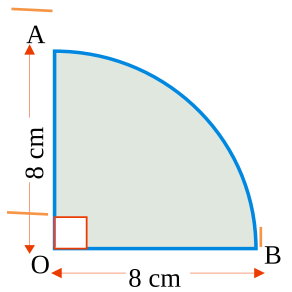


Figure 5.107

Solution: The given figure is $\frac{1}{4}$ of a circle with radius 8 cm.

$A = \frac{1}{4}\ (\pi r^2)$

$A = \frac{1}{4}(3.14 \times (8\text{cm})^2)$

$A = \frac{1}{4}\ (3.14 \times 64\text{cm}^2)$

$A = 50.24\text{cm}^2$

Therefore, the area of the quarter circle is 50.24cm^2.

Example 31. Find the area of each shaded region below.

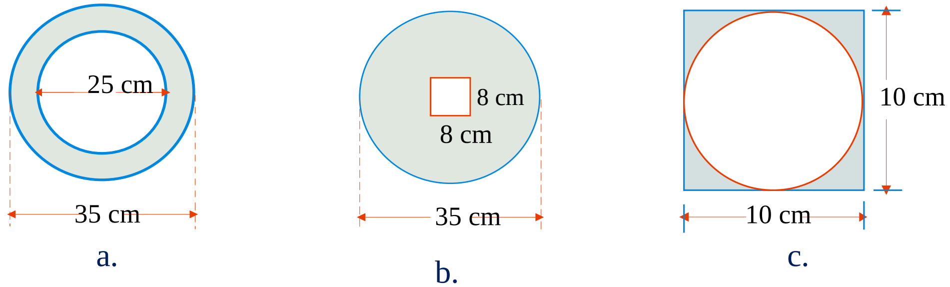


Figure 5.108

Solution:

a. Area of the outer circle $= \dfrac{\pi d^2}{4}$

$= \dfrac{\pi(35\text{cm})^2}{4}$

$= \dfrac{3.14 \times 1225}{4}\text{cm}^2$

$= \dfrac{3846.5}{4}\text{cm}^2$

$= 961.625\ \text{cm}^2$

and area of the inner circle $= \dfrac{\pi d^2}{4}$

$= \dfrac{\pi(25cm)^2}{4}$

$= \dfrac{(3.14 \times 625\text{cm}^2)}{4}$

$= \dfrac{1962.5\text{cm}^2}{4}$

$= 490.625\text{cm}^2$

Therefore, area of the shaded region = area of outer circle – area of inner circle

$= (961.25 - 490.625)\text{cm}^2$

$= 470.625\text{cm}^2$

Therefore, the area of the shaded region is 470.625cm^2.

b. Area of the circle $= \frac{\pi d^2}{4}$

$= \frac{\pi(35\text{cm})^2}{4}$

$= \frac{3.14 \times 1225\text{cm}^2}{4}$

$= \frac{3846.5\text{cm}^2}{4}$

$= 961.625\text{cm}^2$

and area of the square $= S^2$

$= (8\text{cm})^2$

$= 64\text{cm}^2$

Therefore, area of the shaded region = area of a circle – area of a square

$= (961.625 - 64)\text{cm}^2$

$= 897.625\text{cm}^2$

c. Area of the circle $= \frac{\pi d^2}{4}$

$= \frac{3.14 \times (10\text{cm})^2}{4}$

$= \frac{3.14 \times 100\text{cm}^2}{4}$

$= \frac{314\text{cm}^2}{4}$

$= 78.5\text{cm}^2$

and area of the square $= S^2$

$= (10\text{cm})^2$

$= 100\text{cm}^2$

Therefore, area of the shaded region = area of a square – area of a circle

$= (100 - 78.5)\text{cm}^2$

$= 21.5\text{cm}^2$

Hence, the area of the shaded region is 21.5cm^2.

Example 32. If the area of a circle is 154cm^2, then find its circumference $\left(\pi \approx \frac{22}{7}\right)$.

Solution:

i. To find the radius; begin with

$A = \pi r^2$ and put $A = 154\ \text{cm}^2$

Therefore, $\pi r^2 = 154cm^2$

$$\frac{22}{7}r^2 = 154cm^2$$

$$r^2 = 154cm^2 \times \frac{7}{22}$$

$$r^2 = 49cm^2$$

$$r \times r = 7cm \times 7cm$$

$$r = 7cm$$

ii. To find the circumference; use the formula

$C = 2\pi r$ and put $r = 7$ cm,

$$C = 2 \times \frac{22}{7} \times 7cm$$

$$= 44cm$$

Therefore, the circumference of the circle is 44cm.

Example 33. What is the radius of a circle whose curcumfierence is 48πcm.

Solution:

$$C = 2\pi r$$

$$48\pi = 2\pi r$$

Then $r = \frac{48\pi}{2\pi} = 24cm$

Therefore, the radius of the circle is 24cm.

Exercise 5L

1. Find the area of a semicircle whose radius is 2.4 cm.
2. Find the area of a circle if $x = 12$ and $y = 3$, see Figure 5.109 to the right.

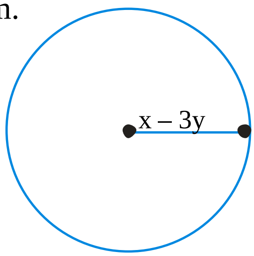


Figure 5.109

3. A square is cut out from a circle as shown in Figure 5.110 to the right. If the radius of the circle is 6cm, what is the total area of the shaded region?

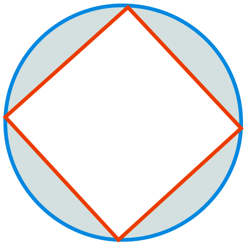


Figure 5.110

4. As shown in Figure 5.111 to the right if the two small semicircles, each of radius 1 unit with centres O′ and O″ are contained in the bigger semi-circle with center O, So that O′, O and O″ are on the same line, then what is the area of the shaded part?

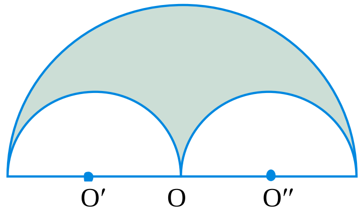


Figure 5.111

5. How many square meters of brick pavement must be laid for a 4meter wide walk around a circular flower bed 22 meters in diameter?
6. If the radius of a circle is twice the radius of another circle, then find the ratio of the areas of the larger circle to the smaller circle.
7. Find the radius of the circle if its area is:

   a. $144\pi cm^2$ b. $324\pi cm^2$ c. $625\pi\ cm^2$ d. $\frac{1}{81}\pi cm^2$

8. Find the diameter of a circle if its area is:

   a. $100\pi cm^2$ b. $16\pi cm^2$ c. $400\pi cm^2$ d. $\frac{1}{4}\pi cm^2$

Challenge Problem

9. In Figure 5.112 the radius of the bigger circle is 9cm, and the area of the shaded region is twice that of the smaller circle, then how long is the radius of the smaller circle?

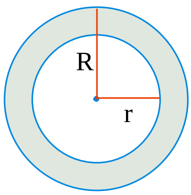


Figure 5.112

10. Two concentric circles (circles with the same centre) have radii of 6cm and 3cm respectively. Find the area of the annulus (the shaded region). (use $\pi = 3.13$)

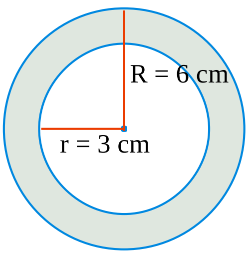


Figure 5.113

5.3.6. Surface Area of Prisms and Cylinder

Group work 5.6

Discuss with your friends.

1. What are the properties of a rectangular prism?
2. How many vertices, lateral edge and lateral faces does a rectangular prism have?
3. Explain why a cube is also a rectangular prism.
4. In Figure 5.114 to the right shows a triangular prism
   a. Use a ruler to construct a net of the solid on plain paper.
   b. Cut out the net and fold it to make the solid.

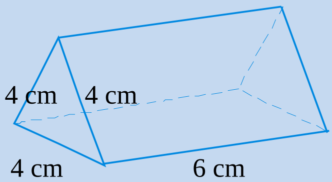


Figure 5.114

5. A, B, C, and D are four solid shapes. E, F, G, and H are four nets. Match the shapes to the nets, (see Figure 5.115).

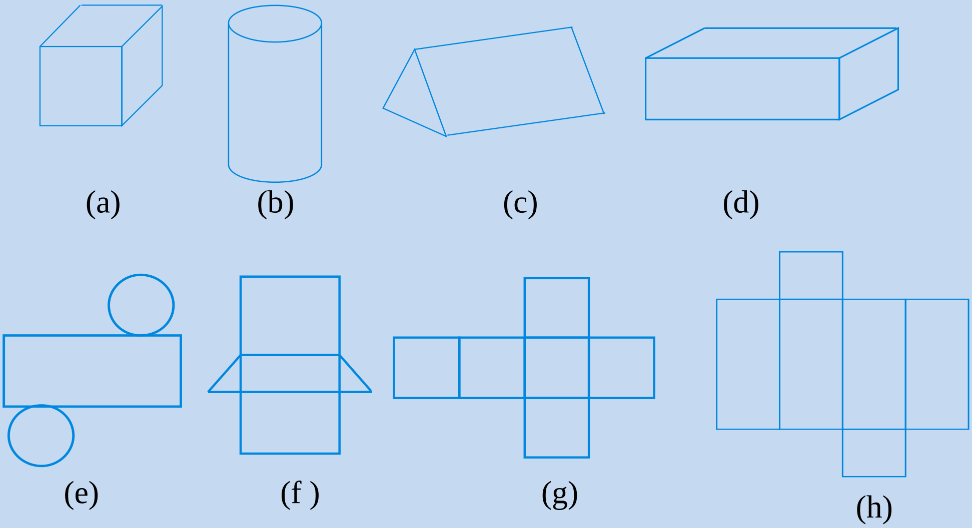
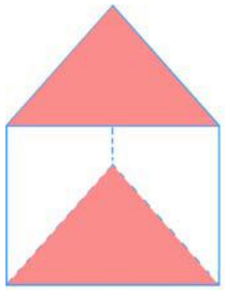


Figure 5.115

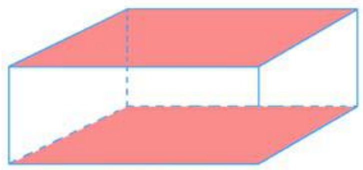
In grade 5 and 6 mathematics lesson you learnt how to compute the volume of a rectangular prisms. In this sub-section you will become more acquainted with these most familiar geometric solids and you will learn how to compute their surface area in a more detailed and systematic ways.

Prisms

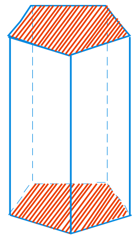
A prism is a solid figure that has two parallel and congruent bases. Depending on the shape of its base a prism can be triangular, rectangular and soon.



Triangular prism (the bases are triangles)



Rectangular prism (the bases are rectangles)



Pentagonal prism (the bases are pentagons)

Figure 5.116

A prism has two bases: upper base and lower base. The edges of a prism are the line segments that bound the prism. Consider the rectangular prism shown in Figure 5.117 to the right.

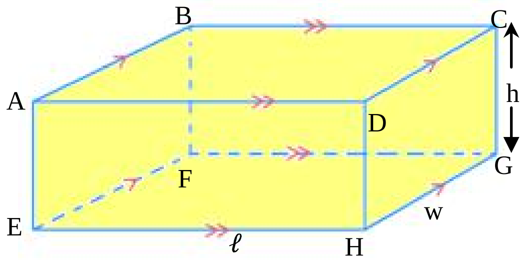


Figure 5.117 rectangular prism

- ✓ The rectangular region ABCD is the **upper base** and rectangle EFGH is the **lower base**.
- ✓ The line segments $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$, $\overline{EF}, \overline{FG}, \overline{GH}$ and $\overline{HE}$ are **edges of the bases** where as $\overline{AE}, \overline{BF}, \overline{CG}$ and $\overline{DH}$ are the **lateral edges** of the prism.
- ✓ The rectangles ABFE, BCGF, CDHG and ADHE are called the **lateral faces** of the prism.

Lateral Surface area of a prism

- ✓ The lateral surface area is the sum of the areas of all lateral faces; denoted by A_S.

$$A_S = a(ABFE) + a(BCGF) + a(CDHG) + a(ADHE)$$
$$= wh + \ell h + wh + \ell h$$
$$= 2\ell h + 2wh$$
$$= 2h(\ell + w)$$
$$= ph \text{ ....Where } p = \text{Perimeter of the base}$$

Total surface area of a prism

A rectanglular prism has six faces: two bases and four lateral faces.

Total surface area = area of the two bases + area of the four lateral faces.

$$A_T = a(ABCD) + a(EFGH) + a(ABFE) + a(BCGF) + a(CDHG) + a(ADHE)$$

$$A_T = \ell w + \ell w + wh + \ell h + wh + \ell h$$

$$A_T = 2\,\ell w + 2wh + 2\,\ell h$$

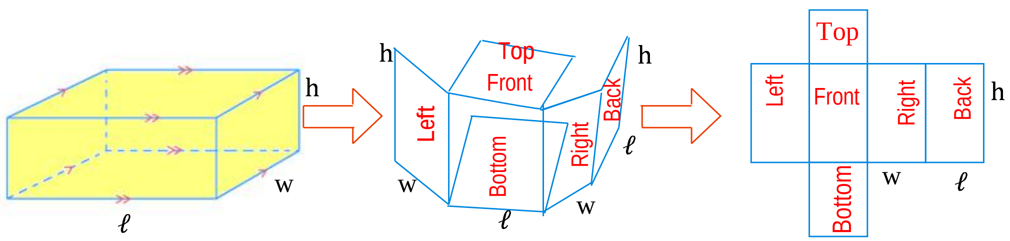
$$A_T = 2(\ell w + wh + \ell h)$$

> **Note:** The total surface area is the sum of the areas of all the faces, denoted by A_T.

? What is a net?

> **Definition 5.17:** A **net** is a pattern of shapes on a piece of paper or card. The shapes are arranged so that the net can be folded to make a hollow solid.

✓ To derive a formula for the surface area of a right prism, we can use the net of the prism. For example consider a rectangular prism in Figure 5.118 below.



Rectangular prism would make Net

Figure 5.118

The surface of a rectangular prism consists of six rectangles. Pair wise these faces or rectangles have equal size, i.e. the front and the back, the right side and the left side and the top and the bottom are rectangles having the sam size.

Thus

- ✓ Area of front face = Area of back face= ℓh
- ✓ Area of left face = Area of right face = wh
- ✓ Area of top face = Area of bottom face= ℓw

The lateral surface area is the sum of the areas of all lateral faces, i.e. lateral surfaces area (A_S) = the sum of the areas of all lateral faces.

or A_S = Area of front face+ Area of back face+ Area of left face+ area of right face.

$= \ell h + \ell h + wh + wh.$

$= 2\ell h + 2wh.$

$= 2h(\ell + w)$

= ph ……..where p = Perimeter of the base.

Total surface area (A_T) = A_S + area of two bases.

$= 2\ell h + 2wh + 2\ell h.$

$= A_S + 2A_B$ …… Where A_B = Area of the base.

Example34. Find the surface area (Total surface area) of the following right rectangular prism.

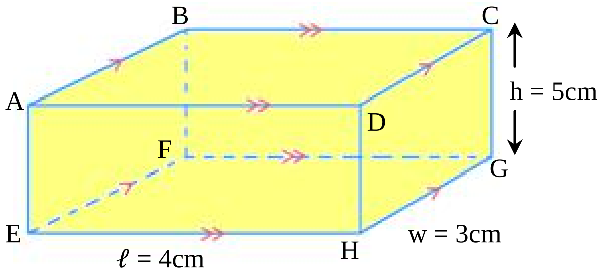


Fig 5.119 Rectangular prism

Method I

Solution:

First find the lateral surface area:

A_S = ABFE + BCGF + CDHG + ADHE

$= (5cm \times 3cm) + (5cm \times 4cm) + (5cm \times 3cm) + (5cm \times 4cm)$

$= 15cm^2 + 20cm^2 + 15cm^2 + 20cm^2$

$= 70cm^2$

Next find the base area:

A_B = EFGH + ABCD

$= 4cm \times 3cm + 4cm \times 3cm$

$= 12cm^2 + 12cm^2$

$= 24cm^2$

Therefore, total surface area(A_T) $= A_S + 2A_B$

$= 70cm^2 + 24cm^2$

$= 94\ cm^2$

Method II

$A_S = 2h(\ell + w)$

$A_S = 2 \times 5cm(4cm + 3cm)$

$A_S = 10cm(7cm)$

$A_S = 70cm^2$

$A_T = 2(\ell w + wh + \ell h)$

$= 2(4cm \times 3cm + 3cm \times 5cm + 4cm \times 5cm)$

$= 2(12cm^2 + 15cm^2 + 20cm^2)$

$= 2(47cm^2)$

$= 94cm^2$

Therefore in both cases (method I and II) we have the same lateral surface area and total surface area, you can use either method I or II but the final answer does not change.

Example 35. Find the surface area(Total surface area) of the following right Triangular prism in which the base is right angled triangle.

Solution:

First find the lateral surface area:

Each base of the prism is a right triangle with hypotenuse 5cm and legs of 3cm and 4cm. Then

A_S = AA'C'C+ C'CBB' + AA'B'B

$= 3cm \times 6cm + 4cm \times 6cm + 5cm \times 6cm$

$= 18cm^2 + 24cm^2 + 30cm^2$

$= 72cm^2$ or As = Ph

$As = (3cm + 4 + 5)^b$

$As = 72\ cm^2$

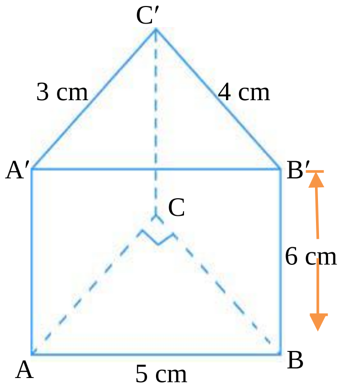


Fig 5.120 Triangular prism

Next find the base area:

$2A_B = a\,(\Delta ABC) + a(\Delta A'B'C')$

$= \frac{1}{2}(3cm \times 4cm) + \frac{1}{2}(3cm \times 4cm)$

$= \frac{1}{2}(12\ cm^2) + \frac{1}{2}(12\ cm^2)$

$= 6\ cm^2 + 6cm^2$

$= 12\ cm^2$

Therefore, total surface area$(A_T) = A_S + 2A_B$

$= 72cm^2 + 12cm^2$

$= 84cm^2$

Cylinders

Definition 5.18: A cylinder is defined as a solid figure whose upper and lower bases are congruent simple closed curves lying on parallel planes.

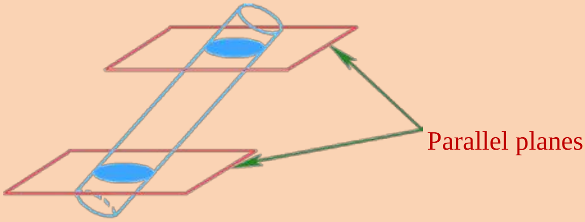


Figure 5.121

Definition 5.19: A right circular cylinder is a cylinder in which the bases are circles and the planes of the bases are perpendicular to the line joining the corresponding points of the bases.

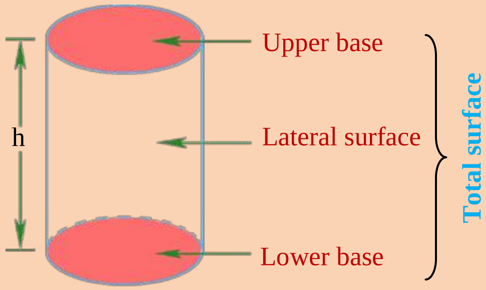


Figure 5.122

Properties of right circular cylinder

1. The upper and the lower bases are congruent (circles of equal radii).
2. The bases lie on parallel planes.
3. A line through the centers of the bases is perpendicular to the diameter of the bases.

Lateral Surface Area of a Cylinder

To calculate the lateral surface area of a circular cylinder, consider a circular cylinder wich is made up of paper. Let us detach the upper and the lower bases, and slit down the side of the cylinder as shown in Figure 5.123 below.

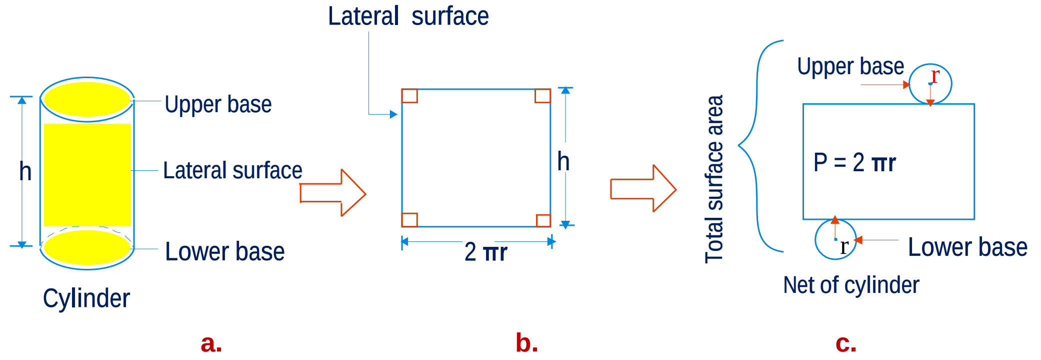


Figure 5.123

The upper and lower bases of the cylinder are **parallel** and **congruent**. Therefore, they have equal area: $\mathbf{A_B = \pi r^2}$. If the upper and the lower bases are detached, then you get **a rectangle** whose length is **$2\pi r$** and height **h** which is the height of the cylinder.

Therefore, the lateral surface area $\mathbf{(A_S) = 2\pi rh}$ or As = Ph, P = C

$$\mathbf{As = Ch}$$

$$\mathbf{As = 2\pi\ rh}$$

In general, for any circular cylinder,

1. The area of the bases: $2A_B = 2\pi r^2$.
2. The area of the lateral surface $(A_S) = 2\pi rh$.
3. The total surface area of the cylinder whose base radius r is:

   $A_T = 2A_B + A_S$

   $A_T = 2\pi r^2 + 2\pi rh$

   $A_T = 2\pi r\ (r + h)$

Example 36. The radius of the base of a right circular cylinder is 4cm and its height is 6cm. Find the total surface area of the cylinder in terms of π.

Solution:

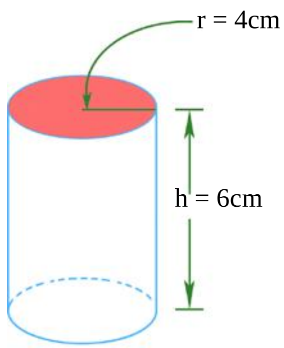


Figure 5.124 cylinder

See Figure 5.124.

$A_S = 2\pi rh$

$A_S = 2\pi(4cm \times 6cm)$

$A_S = 48\pi cm^2$

Therefore, the lateral surface area is $48\pi cm^2$

$2(\text{Base area}) = 2\pi r^2$

$2A_B = 2\pi(4cm)^2$

$= 32\pi cm^2$Total surface area (A_T)

$= A_S + 2A_B$

$= (48\pi+32\pi)cm^2$

$= 80\pi cm^2$

Therefore, the total surface area of the cylinder is $80\pi cm^2$

Example 37. The sum of the height and radius of a right circular cylinder is 9m. The surface area of the cylinder is $54\pi m^2$. Find the height and the radius.

Solution:

Let the height of the cylinder be h and the radius of the base be r

This implies, $h + r = 9m$

$h = 9 - r$

$A_T = 54m^2$ …………… Given

$A_T = 2\pi rh + 2\pi r^2$ ….. Given formula

$54\pi = 2\pi r(9 - r) + 2\pi r^2$ ……Substitute h by 9 – r as h = 9 – r

$54\pi = 18\pi r - 2\pi r^2 + 2\pi r^2$

$54\pi = 18\pi r$

$r = 3m$

Therefore, the radius of the cylinder is 3m.

Thus $h = 9 - r$

$h = 9m - 3m$

$h = 6m$

Therefore, the height of the cylinder is 6m.

Exercise 5M

1. If the edge of a cube is 4cm, then find:
   a. its lateral surface area.
   b. its total surface area.
2. A closed cardboard box is a cuboid with a base of 63cm by 25cm. The box is 30cm heigh, calculate the total surface area of the box.
3. The lateral surface area of a right circular cylinder is $120cm^2$ and the circumference of the bases is 12cm. Find the altitude of the cylinder.
4. The total surface area of a right circular cylinder is $84\pi cm^2$ and the altitude is 11cm. Find the radius of the base.

Challenge Problem

5. In Figure 5.125 to the right find:
   a. Lateral surface area
   b. Total surface area

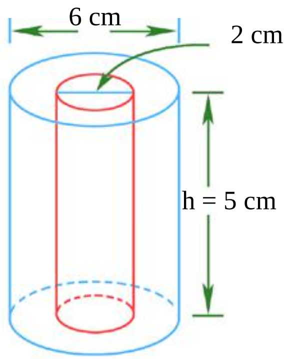


Figure 5.125

5.3.7. Volumes of prism and cylinder

Group Work 5.7

1. **A box in the shape of a cuboid has a volume of 50cm^3. It has a length of 8cm and a height of 2.5cm. What is its width.**
2. **A right circular cylinder has a height of 20cm and a diameter of 6cm. what is its volume?**
3. **A right triangular prism has height 12cm and volume 60cm^3. What is the area of the triangular bases?**

In Grade 5 and 6 mathematics lesson you learnt how to compute the volume of prisms. In this lesson you will learn how to compute the volume in a more detailed and systematic ways.

The volume of a solid geometric figure is a measure of the amount of space it occupies. Most commonly used units of volume are cubic centimeters (cm^3) and cubic metres (m^3).

Volume of Prism

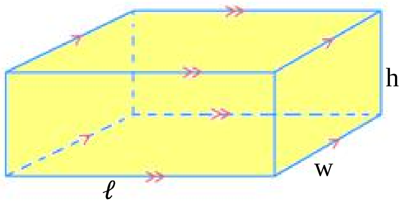


Fig 5.126 Rectangular prism

1. The volume (V) of a rectangular prism equals the product of its length (ℓ), width (w) and height (h). That is, volume of rectangular prism
   = length × width × height
   Volume: $V = \ell \times w \times h$
2. In a cube the length, width and height are all the same size. So the formula for the volume is:

   Volume of a cube = length × length × length
   $= \ell \times \ell \times \ell$
   $= \ell^3$

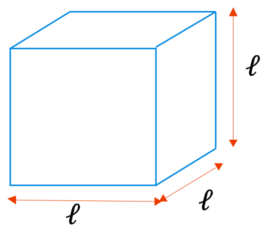


Figure 5.127 cube

3. The Volume (V) of a right triangular prism equals the product of its base area and its height. That is, volume of a right triangular prism = Base Area × height.

   Volume: $V = A_B \times h$

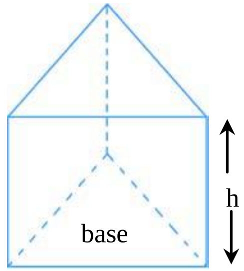


Figure 5.128 Triangular prism

4. The volume of any prism equales the product of its base area and altitude. That is,

   $V = A_Bh$ where A_B = base area and
   h = height

Example 38. Shown in Figure 5.129 to the right. Find the volume of the rectangular prism.

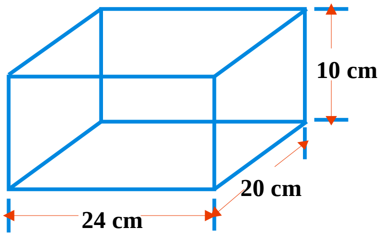


Figure 5.129 Rectangular prism

Solution:

$$V = A_Bh$$
$$= (24\text{cm} \times 20\text{cm}) \times 10\text{cm}$$
$$= 4800\text{cm}^3$$

Therefore, the volume of the rectangular prism is 4800cm^3.

Example 39. Find the volume of the triangular prism, Shown in Figure 5.130 below.

Solution:

$$V = A_Bh$$

But the area of the base is

$$A_B = \frac{1}{2}ab$$
$$= \frac{1}{2}(4\text{cm} \times 3\text{cm}) = 6\text{cm}^2$$

Hence $V = 6\text{cm}^2 \times 8\text{cm}$

$V = 48\text{cm}^3$

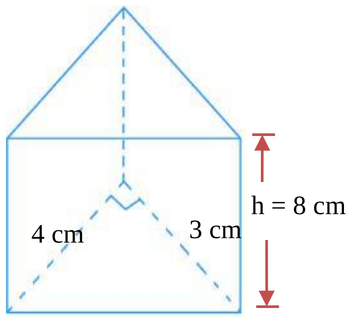


Figure 5.130 Rectangular prism

Therefore, the volume of the triangular prism is 48cm^3.

Volume of Cylinder

A circular cylinder is a special prism where the base is a circle. The area of the base with radius r is πr^2, so its volume V = area of the base × height

$$V = \pi r^2 \times h = \pi r^2 h.$$

Volume of a cylinder = base area × height

Volume: $V = A_B h$

$V = \pi r^2 h$

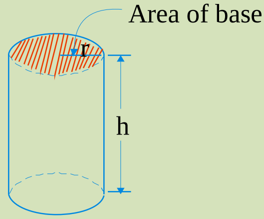


Figure 5.131

Example 40. Find the volume of a circular cylinder shown in Figure 5.132 below. Leave your answer interms of π.

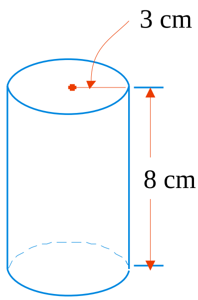


Figure 5.132

Solution:

$V = \pi r^2 h$

$V = \pi(3cm)^2 \times (8cm)$

$V = 72\pi cm^3$

Therefore, the volume of the cylinder is $72\pi cm^3$.

Example 41. The volume of a circular cylinder is $48\pi cm^3$. Find the height of this cylinder, if its base radius is 2 cm.

Solution:

$V = 48\pi cm^3$

$r = 2cm$

$V = \pi r^2 h$

Then $h = \frac{V}{\pi r^2}$

$h = \frac{48\pi}{\pi(2)^2} = \frac{48\pi}{4\pi} = 12$

Therefore, the height of the cylinder is 12cm.

Exercise 5N

1. The container in Figure 5.133 is made from a circular cylinder and a cube. The height of the cylinder is 20 cm and its base ralius is 8 cm. The cube has sides of 16cm.
   a. Calculate the volume in cm^3, of the cylinder.
   b. Calculate the total volume in cm^3 of the container.

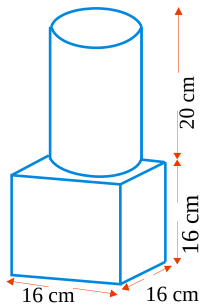


Figure 5.133

2. The volume of a triangular prism is $204 cm^3$. If its height is 24 cm, then find the area of its base.

3. Calculate the volume of the triangular prism Given in Figure 5.134 to the right .

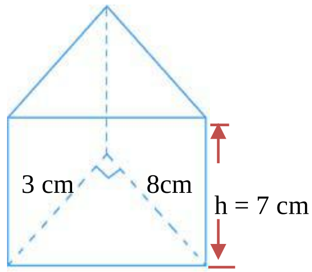


Figure 5.134 Triangular prism

4. Find the volumes of these solids.

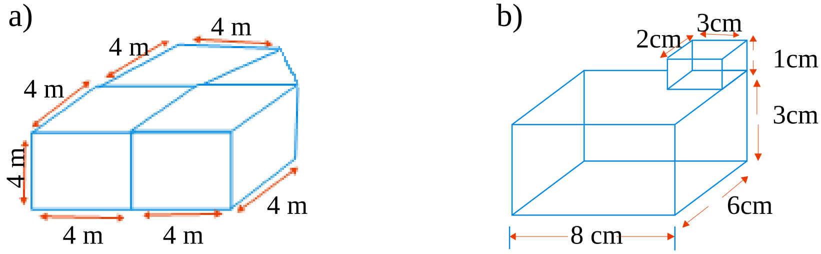
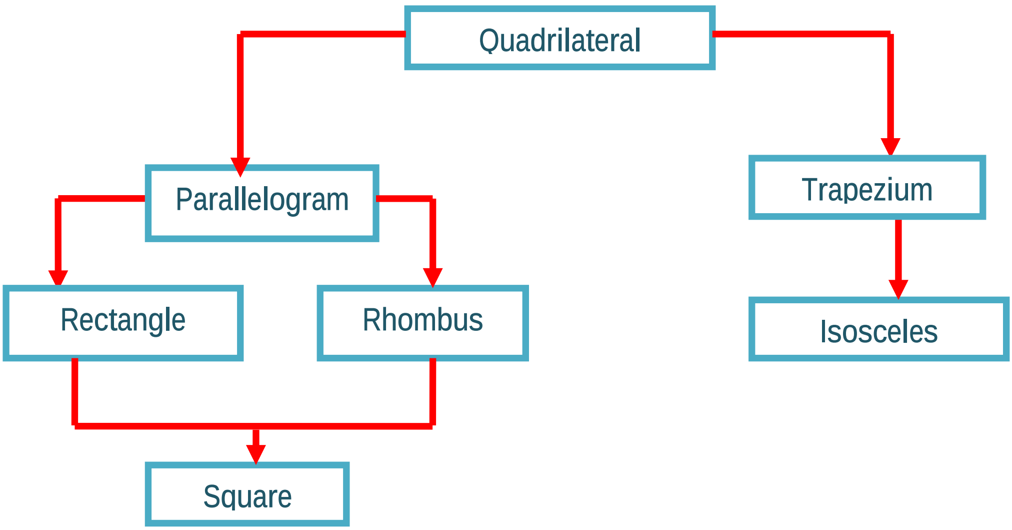


Figure 5.135

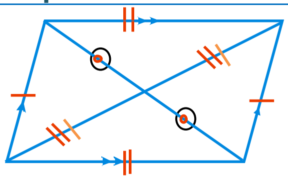
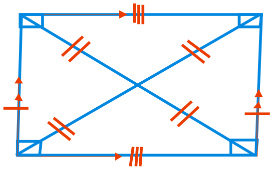
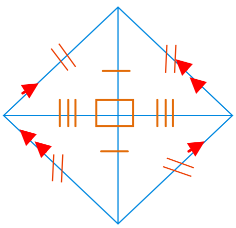
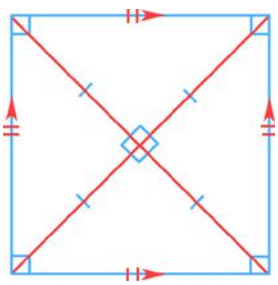
Summary For unit 5

1. In general quadrilateral can be classified as follows:



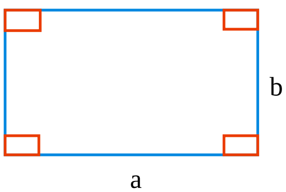
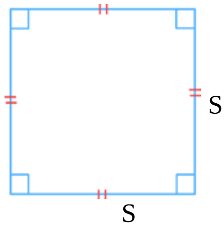
2. Aquadrilateral is a four-sided geometric figure boundad by line segments.

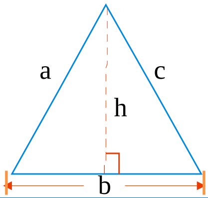
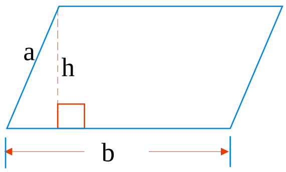
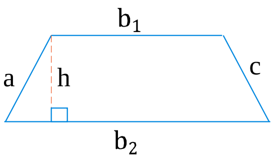
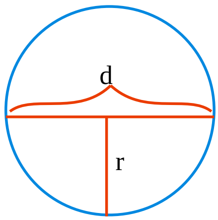
3. Table 5.4

Name	shape	Properties
Parallelogr am		- Opposite sides are congruent. - Opposite angles are congrunet. - The diagonals bisect each other. - Two consecutive angles are supplementary. - Opposite sides are parallel.
Rectangle		- Both pairs of opposite sides are parallel. - All angles are right angles. - The diagonals are congruent. - Both pairs of opposite sides are congruent.
Rhombus		- All sides are congruent. - The diagonals cut at right angles. - The angles are bisected by the diagonals. - Both pairs of opposite sides are paralles. - Opposite angles are congrunet.
Square		- All sides are congruent. - All angles are right angles. - The diagonals are equal and bisect each other at right angle. - Each diagonal of a square makes an angle of 45^{o} with each side of the square. - Both pairs of opposite sides are parallel.

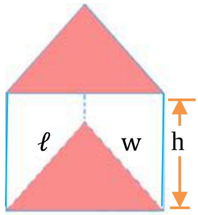
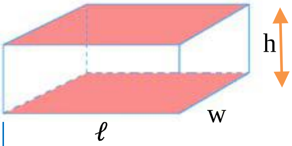
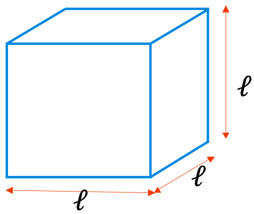
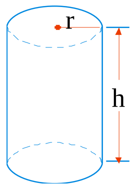
4. A **polygon** is a simple closed path in a plane which is entirely made up of a line segment joined end to end.

5. A **convex polygon** is a simple polygon in which each of the interior angles measure lessthan 180^o.
6. A **concave polygon** is a simple polygon which has at least one interior angle of measure greater than 180^o.
7. A diagonal of a **convex polygon is** a line segment whose end points are non – consecutive vertices of the polygon.
8. A **circle** is the set of all points in a plane that are equidistant from a fixed point called the **center of the circle**.
9. A **chord of a circle** is a linesegment whose end points are on the circle.
10. A **diameter of a circle** is any chord that passes through the center and denoted by ‘d’.
11. A **radius of a circle** is a linesegment that has the center as one end point and a point on the circle as the other end point and denoted by ‘r’.
12. The formula for the number of triangle, (T) determined by diagonals drawn from one vertex of a polygon of n sides is $T = n – 2$.
13. A polygon which is both equilateral and equiangular is called a **regular polygon**.
14. The sum S of the measures of all the interior angles of a polygon of an sides is given by $S = (n – 2)\ 180^o$.
15. The measure of each interior angle of n – sided regular polygon is $\frac{(n-2)180^o}{n}$.
16. Table 5.5

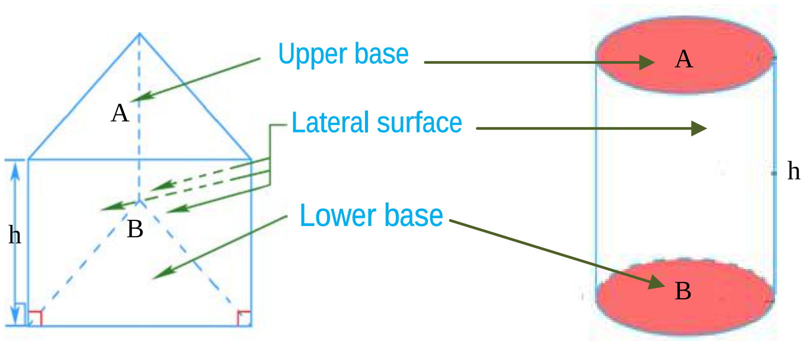
Name	Shape	Area	Perimeter (circumference)
Rectangle		$A = a \times b$	$P = 2(a + b)$
Square		$A = S^2$	$P = 4S$

Triangle		$A = \frac{1}{2}bh$	$P = a + b + c$
Parallelogram		$A = bh$	$P = 2(a + b)$
Trapezium		$A = \frac{h}{2}(b_1 + b_2)$	$P = a + b_1 + c + b_2$
Circle		$A = \pi r^2$ $= \pi\left(\frac{d}{2}\right)^2$ $= \frac{\pi d^2}{4}$	$C = 2\pi r$ $= \pi d$

17. **A geometric solid figure** is said to be right prism, if the parallel planes containing the upper and lower bases and any line on the lateral edge makes right angle with the edge of the base.
18. **A net** is a pattern of shapes (rectangles, triangles, circles or any shapes) on a piece of paper (or card) and when correctly folded gives a model of solid figure
19. **A right circular cylinder** is a cylinder in which the bases are circles and aline through the two centers is perpendicular to radius of the bases.
20. Table 5.6 Here A_S = area of lateral surface; A_B = Base area, P = perimeter of the base and A_T = Total surface Area.

Name	Shape	Area	Volume
Triangular prism		$A_S = 2h(\ell + w)$ $= ph$ $A_T = A_S + 2A_B$ $A_T = Ph + 2A_B$	$V = A_B h$
Rectangular prism		$A_S = 2h(\ell + w)$ $= Ph$ $A_T = 2(\ell w + wh + \ell h)$ Or $A_T = A_S + 2A_B$ $A_T = Ph + 2A_B$	$V = A_B h$
Cube		$A_S = 4\ell^2$ $A_T = 6\ell^2$	$V = \ell^3$
Circular Cylinder		$A_S = 2\pi rh$ $A_T = 2\pi r(r + h)$	$V = A_B h$ $= \pi r^2 h$

21.



Right prism

Right circlar cylinder

Figure 5.136

Miscellaneous EXERCISE 5

I. Write true for the correct statements and false for the incorrect ones.

1. Every rectangle is asquare.
2. Every rhombus is a rectangle.
3. Atrapzium is a parallelogram.
4. The diagonal of a parallelogram divides the parallelogram in to congruent triangles.
5. Every square is a rectangle.
6. The angles of a rectangle are congruent.
7. All the sides of a parallelogram are congruent.

II. Choose the correct answer from the given four alternatives.

1. In Figure 5.137 below the two lines ℓ_1 and $\ell_2$are parallel where t_1 and t_2 are transuersal lines. What is the measure of the angle marked z?

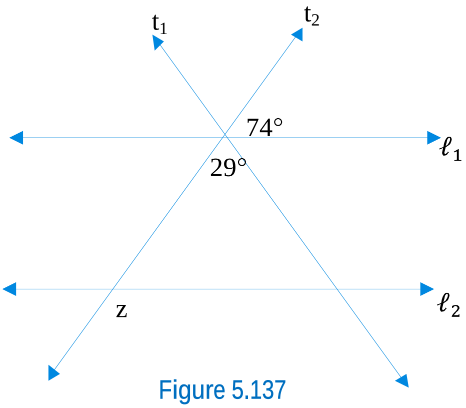


Figure 5.137

a. 72^o
b. 29^o
c. 103^o
d. 106^o

2. The sum of the measures of the interior angles of a polygon is 900^o. How many sides does the polygon have?

a. 5 b. 6 c. 7 d. 8

3. If the sum of the three angles of a quadrilateral is equal to $1\frac{1}{2}$ times the sum of the three angles of a triangle, what is the measure of the fourth angle of the quadrilateral?

a. 90^o b. 60^o c. 45^o d. 30^o

4. **Which of the following plane figures have always a perpendicular diagonals?**
   a. Rectangle **c. Rhombus**
   b. Trapezium **d. parallelogram**
5. **The angles of a triangle are in the ratio of 2:3:5. What is the size of the largest angle in degrees?**
   a. 90^o **b. 110^o** **c. 45^o** **d. 72^o**
6. **Which of the following statements is not true?**
   a. The diagonal of a rectangle is longer than any of its sides.
   b. If a parallelogram has equal diagonals, then it is a square.
   c. The diagonals of a rhombus divide the rhombus in to four right angled triangles of equal area.
   d. Diagonals of a parallelogram bisect each other.
7. **2x, 3x, 4x, 11x and 7x are measures of the interior angles of five sided convex plygon, what is the measure of the largest angle in degrees?**
   a. 200 **b. 270** **c. 366** **d. 220**
8. **In Figure 5.138 below, the value of x is:**
   a. 60^o
   b. 40^o
   c. 50^o
   d. 65^o

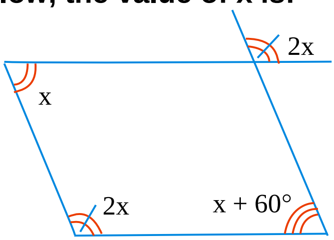


Figure 5.138

9. **In Figure 5.139 below, which one of the following is false?**
   a. $x = 36^o$
   b. $y = 152^o$
   c. $w = 2x$
   d. $k = 144^o$

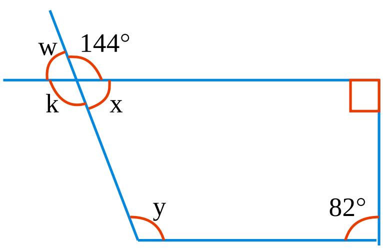


Figure 5.139

10. **In Figure 5.140 below, if $\overline{PQ}$ and $\overline{SR}$ are parallel lines, then which one of the following is false.**

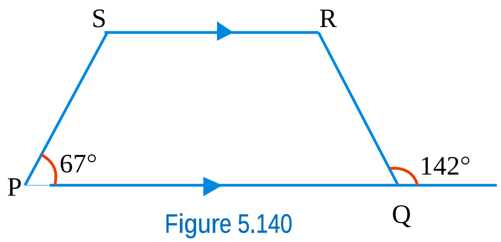


Figure 5.140

a. **m(∠PSR) = 113°**

b. **m(∠QRS) = 142°**

c. **m(∠PQR) = 38°**

d. **m(∠PSR) = 38°**

11. **In Figure 5.141, the value of n is:**

a. **57.5°**

b. **65°**

c. **50°**

d. **130°**

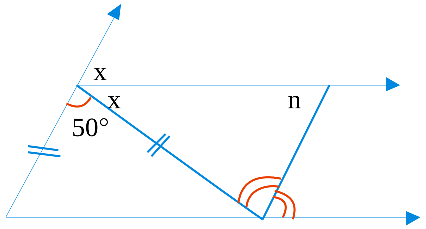


Figure 5.141

12. **Which expression describes the area of the shaded region?**

a. **$\pi(R+r)(R-r)$**

b. **$\pi R^2 - \pi r^2$**

c. **$\pi(R^2 - r^2)$**

d. **all are correct**

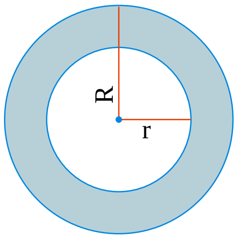


Figure 5.142

13. **What is the total surface area of a right triangular prism whose altitude is 15cm long and whose base is a right angled triangular with lengths of sides 6cm, 8cm and 10cm?**

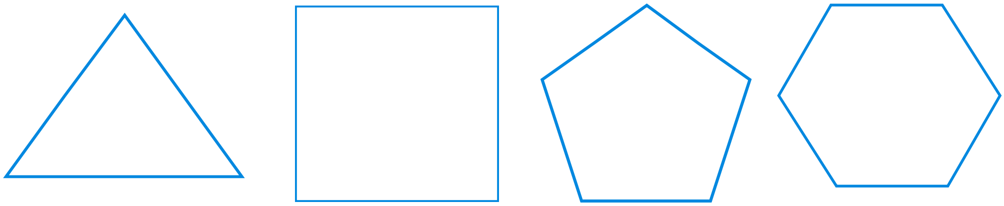
a. **$360cm^2$** **b. $408cm^2$** **c. $420cm^2$** **d. $440cm^2$**

14. **The altitude and the radius of the base of a right circular cylinder are equal. If the lateral surface area of the cylinder is $72\pi cm^2$, then the length of the altitude is:**

a. **$2\sqrt{9}$ cm** **b. $6\sqrt{2}$ cm** **c. 6cm** **d. 36cm**

III. Work out problems

15. Trace all these shapes:



a) b) c) d)

Figure 5.143

16. Draw all the diagonals in each shape. (Make sure each vertex is joined to every other vertex) (use Figure 1.143).

17. Copy this Table 5.7 and fill it in for each shape. (use Figures 5.143)

	Name	Number of sides	Number of vertices	Number of diagonals
a				
b				
c				
d				

18. In Figure 5.144, of ΔABC, where $m(\angle C) = 30°$, $m(\angle ABD) = 5x$, $m(\angle A) = 4x$. Find $m(\angle ABC)$ in degrees.

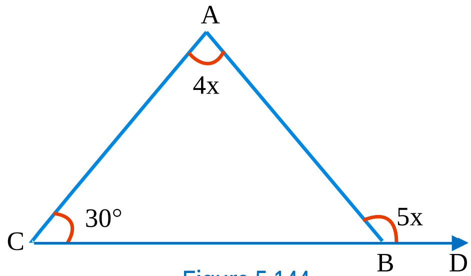


Figure 5.144

19. The measure of each interior angle of a regular polygon with n sides is given by the formula: $\left(\frac{2n-4}{n}\right) \times 90^{o}$.

Calculate the measure of each interior angle of a regular polygon with

a. 30 sides **b. 45 sides** **c. 90 sides**

20 **In Figure 5.145 below, find m(∠DBC) and m(∠CAD).**

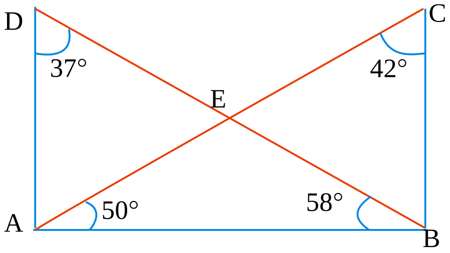


Figure 5.145

21 **In Figure 5.146, m(∠D) = 112°, $\overline{DA}$ bisects (∠CAB), $\overline{DB}$ bisects (∠CBA). Find m(∠C).**

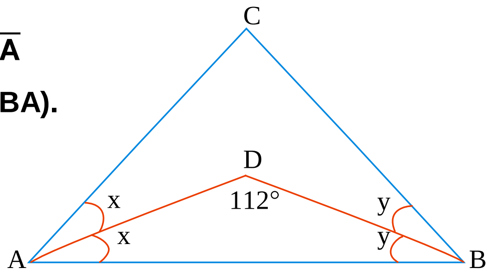


Figure 5.146

22 **The base of a right prism is an equilateral triangle each of whose sides measures 4cm. The altitude of the prism measures 5cm. Find the volume of the prism.**

23 **The circumference of a circle is 60cm. Calculate:**

- **a. the radius of the circle.**
- **b. the area of the circle.**

24 **The volume of a right circular cylinder is 1540cm^3 and its altitude is 10cm long. What is the length of the radius of the base? (Take $\pi = \frac{22}{7}$)**

25 **Find the height of a right circular cylinder whose volume is 75cm^3 and base radius is $\frac{50}{2}$mm.**